Math 2410-004/013 **Name:** Solutions **Due:** 2/9/18

Homework #3: Sections 2.1

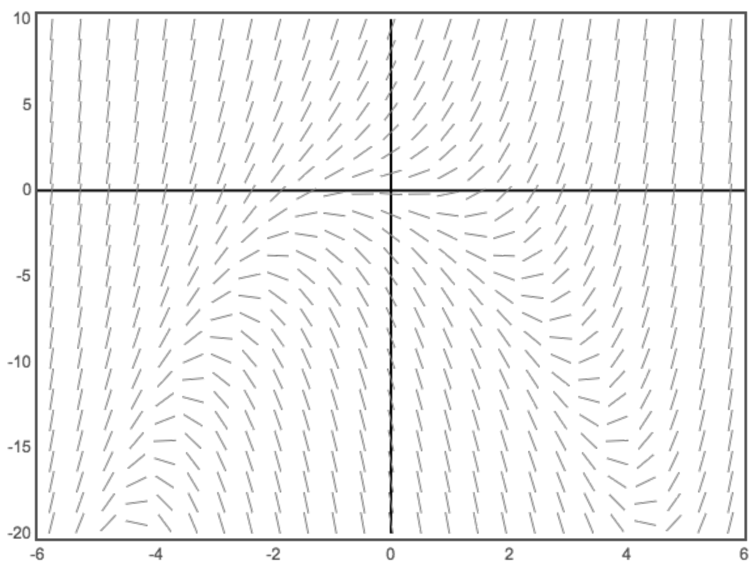
1. The direction field for

$$\frac{dy}{dx} = x^2 + y$$

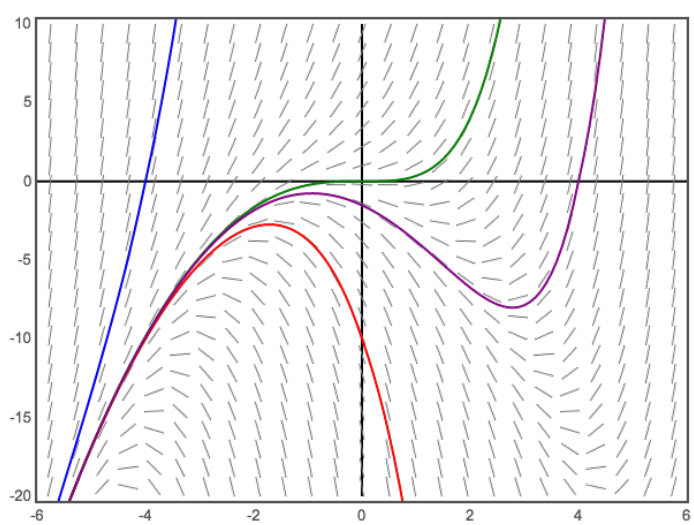
is given below. Consider the following initial conditions

$$(a)\ y(0) = -10 \qquad (b)\ y(0) = 0 \qquad (c)\ y(-4) = 0 \qquad (d)\ y(4) = 0$$

For each condition, sketch a solution curve to the differential equation that satisfies that condition.

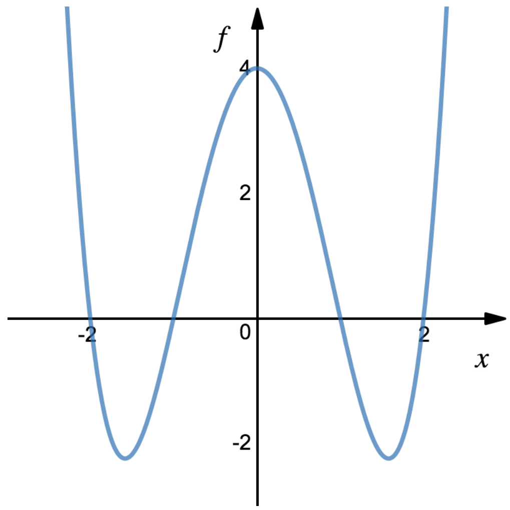


Solution: Plotted below are the desired solution curves. The red curve satisfies (a). The green curve satisfies (b). The blue curve satisfies (a). The purple curve satisfies (d).

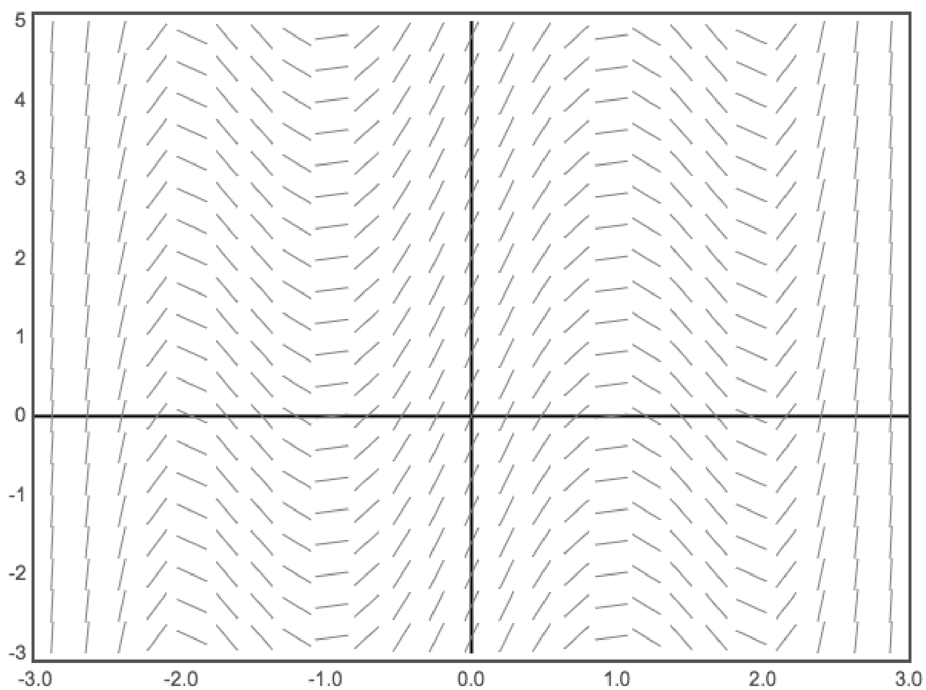


2. The function $f(x)$ is plotted below. By hand, sketch a direction field over an appropriate grid for the DE

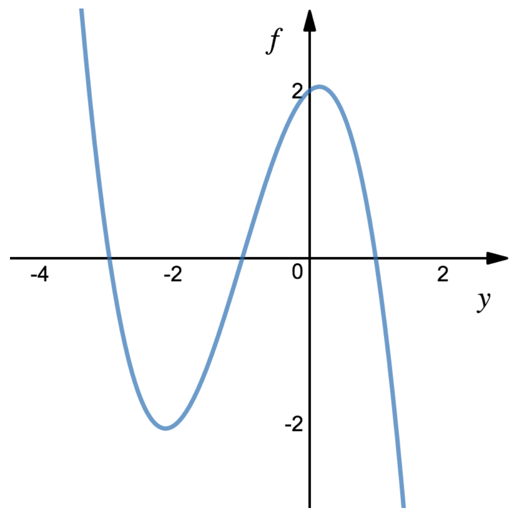
$$\frac{dy}{dx} = f(x)$$



Solution: Recognizing that the differential equation depends only on x allows us to conclude lineal elements along the same vertical line ($x = x_0$) will be parallel. Estimating the values of $f(x)$ via the graph allow us to find the appropriate slopes along the x-axis and we extrapolate up and down.



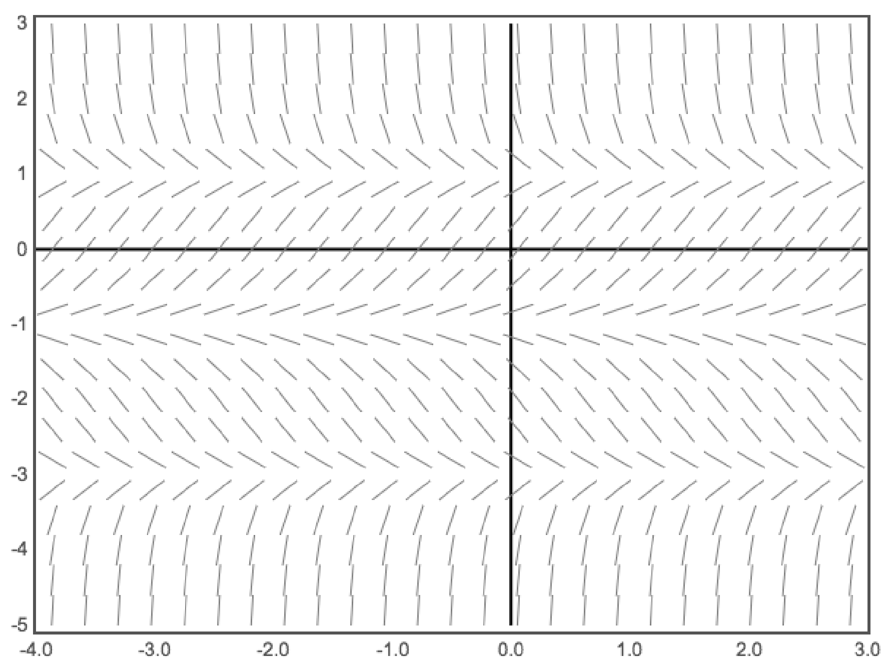
3. The function $f(y)$ is plotted below.



(a) Sketch a direction field over an appropriate grid for the DE

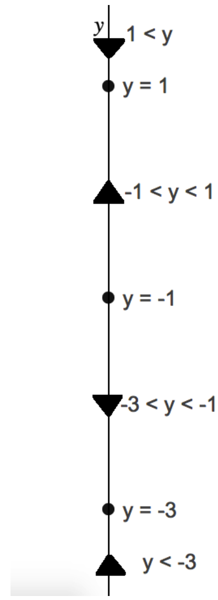
$$\frac{dy}{dx} = f(y) \tag{1}$$

Solution: Recognizing that the differential equation depends only on y allows us to conclude lineal elements along the same horizontal line ($y = y_0$) will be parallel. Estimating the values of $f(y)$ via the graph allow us to find the appropriate slopes along the y-axis and we extrapolate left and right.



(b) Use the graph to locate the critical points of and sketch a phase portrait 1.

Solution: The critical points of the DE are where $f(y) = 0$. Clearly, these are at $y = -2$, $y = -1$ and $y = 1$. Noticing that $f(y) > 0$ for $y < -2$, that $f(y) < 0$ for $-3 < y < -1$, that $f(y) > 0$ and $-1 < y < 1$ and that $f(y) < 0$ for $y > 1$ allows us to sketch the phase portrait below.



(c) Sketch typical solution curves in the subregions in the xy-plane determined by the graphs of the equilibrium solutions. (Make sure to include the equilibrium solutions!)

Solution: Seeing that $f(y)$ is continuously differentiable allows us to conclude that the solution curves will tend towards or away from the nearest equilibria based on the phase portrait. Plotting solutions in each of these regions yields the following plot.