Math 2410-004/013 **Name:** Solutions **Due:** 1/26/18

Homework #1: Section 1.1

1. (a) What is a differential equation? Give an example different from the one in lecture.

Solution: A differential equation is simply an equation which involves derivatives. Here is an example:

$$\frac{d^{27}y}{dx^{27}} = \frac{d^{943}y}{dx^{943}} + \cosh(y + x)$$

(b) Define the order of a differential equation. Give an example of a second order differential equation as well as an example of a higher order differential equation.

Solution: The order of a differential equation is simply the order of the highest derivative present in the equation. For example,

$$y'' = y \text{ and } y^{(5)} = x$$

are differential equations of order 2 and 5 respectively.

2. (a) Define a linear differential equation. Give an example of both a linear and nonlinear differential equation different from the ones in lecture.

Solution: A linear differential equation is one who is linear in the dependent variable as well as its derivatives. For example

$$\sin(x)y'' + e^x y' + \ln(\sin(\arctan(x^2)))y = x^2$$

is a linear differential equation, while

$$y'' \cdot y = 1$$

is a nonlinear differential equation.

(b) What is a solution to a differential equation? Verify that both $y = e^{3x}$ and $y = 2e^{3x}$ are solutions of the following differential equation

$$\frac{dy}{dx} = 3y.$$

Solution: A solution to a differential equation is, roughly, a function which satisfies the equation. More precisely, it is a function defined on an interval with n continuous derivatives on that interval which all satisfy the nth order differential equation.

Here e^{3x} and $2e^{3x}$ are both infinitely continuously differentiable on the entire real line so there are no worries about the intervals of definition for these two functions. To see that they satisfy the DE in question, and are hence solutions observe the following. Letting $y_1 = e^{3x}$ and $y_2 = 2e^{3x}$, we have

$$\frac{dy_1}{dx} = \frac{d}{dx}e^{3x} = 3e^{3x} = 3y_1$$
$$\frac{dy_2}{dx} = \frac{d}{dx}2e^{3x} = 6e^{3x} = 3(2e^{3x}) = 3y_2$$

Note: The formal definitions from the 1.1 handout are also suitable as answers to each of these questions. Here I tried to provide intuitive answers that will serve you as we move forward.

3. **Bonus:** (2 points) Describe how differential equations arises in either your major or one of your academic interests.

> **Solution:** I study a sub-area of mathematical logic tasked with determining the necessary axioms to prove particular theorems in mathematics. The theory of differential equations if full of such theorems to analyze in this manner. We've already seen one!