Math 2410-004/013 **Name:** ______________________________ **Due:** N/A

Homework #4: Sections 2.6, 2.3, 2.5

1. Use Euler's method with step size $h = 0.1$ to approximate $y(1.2)$, where $y(x)$ is the solution to the IVP

$$\frac{dy}{dx} = 1 + x\sqrt{y}, \quad y(1) = 9.$$

2. Consider the piecewise defined linear differential equation

$$\frac{dy}{dx} + 2xy = f(x), \text{ where } f(x) = \begin{cases} x & \text{if } 0 \leqslant x < 1 \\ 0 & \text{if } x \geqslant 1 \end{cases}$$

Here we will find a continuous function $y(x)$ which satisfies this differential equation as well as the initial condition that $y(0) = 2$.

(a) Solve the IVP

$$\frac{dy}{dx} + 2xy = f(x), \qquad y(0) = 2$$

on the interval $[0, 1)$. Call this solution $y_1(x)$.

(b) Find a family of solutions to the differential equation

$$\frac{dy}{dx} + 2xy = f(x)$$

on the interval $(1, \infty)$. Use c for your constant of integration and call this family of solutions $y_2(x)$.

(c) Using your answers from the previous two parts, define the piecewise function

$$y(x) = \begin{cases} y_1(x) & \text{if } 0 \leqslant x < 1 \\ y_2(x) & \text{if } x \geqslant 1 \end{cases}$$

Find a value of c such that this function is continuous.

(d) Discuss why it is not fit to formally refer to $y(x)$ as a *solution* on $[0, \infty)$ but instead, simply as a continuous function which satisfies the differential equation on $[0, \infty)$.
It may help to plot $f(x)$ and $y(x)$ over the interval $(0, 5)$.

3. By using an appropriate substitution, solve the following differential equation

$$\frac{dy}{dx} - y = e^x y^2.$$