Math 2410-004/013 **Name:** ____________________________ **Due:** 2/2/18

Homework #2: Sections 1.2, 1.3, 2.2

1. (a) Verify that $3x^2 - y^2 = c$ is a one-parameter family of implicit solutions of the differential equation $y\dfrac{dy}{dx} = 3x$.

   (b) Sketch the graph of the particular solution $3x^2 - y^2 = 3$. Find all explicit solutions $y = \varphi(x)$ of the DE in part (a) defined by this relation (whose interval of definition is as large as possible.) Specify the interval of definition I for each such solution.

   (c) The point $(-2, 3)$ is on the graph of $3x^2 - y^2 = 3$. Which of the solutions you found in part (b) satisfy the IVP

$$y\frac{dy}{dx} = 3x \qquad y(-2) = 3.$$

2. Suppose that the first-order differential equation $y' = f(x, y)$ possesses a one-parameter family of solutions and that $f(x, y)$ satisfies the hypotheses of the existence and uniqueness theorem on some rectangle R in the xy-plane. Explain why two different solution curves cannot intersect or be tangent to each other at a point (x_0, y_0) in R.

3. Suppose that a large mixing tank initially holds 300 gallons of water in which 50 pounds of salt have been dissolved. Pure water is pumped into the tank at a rate of 3 gallons every minute and when the solution is well stirred, it is then pumped out at the same rate.

   (a) Determine a differential equation for the amount of salt $A(t)$ in the tank at time $t > 0$.

   (b) Determine $A(0)$ and use this as an initial condition to construct an IVP using the differential equation found in part (a). Solve this IVP.

   (c) At what time will the concentration of salt in the solution be half of it's initial value?

   (d) **Bonus:** (2 points) Will the tank ever be free of salt? Discuss your answer in terms of limits.

4. Solve the following IVP:

$$t^2\frac{dx}{dt} = x - tx \qquad x(-1) = -1.$$