The Laplace transform

Math 2410-004/013

April 23, 2018

Recall that the Laplace transform of a function $f(t)$ is defined to be

$$\mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t)\, dt = F(s)$$

for all s such that the integral converges.

Below you will find a table of common Laplace transforms and inverse Laplace transforms. Further below is a table summarizing some of the properties of the Laplace transform.

$f(t) = \mathcal{L}^{-1}[F(s)]$	$F(s) = \mathcal{L}[f(t)]$	$f(t) = \mathcal{L}^{-1}[F(s)]$	$F(s) = \mathcal{L}[f]$
$f(t) = e^{at}$	$F(s) = \dfrac{1}{s-a}$ for $s > a$	$f(t) = t^n$	$F(s) = \dfrac{n!}{s^{n+1}}$ for $s > 0$
$f(t) = \sin(\omega t)$	$F(s) = \dfrac{\omega}{s^2+\omega^2}$	$f(t) = \cos(\omega t)$	$F(s) = \dfrac{s}{s^2+\omega^2}$
$f(t) = \mathcal{U}(t-a)$	$F(s) = \dfrac{e^{-as}}{s}$ for $(s > 0)$		

Rules for Laplace transforms	Rules for inverse Laplace transform
$\mathcal{L}[f'(t)] = s\mathcal{L}[f(t)] - f(0)$	
$\mathcal{L}[f''(t)] = s^2\mathcal{L}[f(t)] - sf(0) - f'(0)$	
$\mathcal{L}[af_1(t) + bf_2(t)] = a\mathcal{L}[f_1(t)] + b\mathcal{L}[f_2(t)]$	$\mathcal{L}^{-1}[aF_1(s) + bF_2(s)] = a\mathcal{L}^{-1}[F_1(s)] + b\mathcal{L}^{-1}[F_2(s)]$
$\mathcal{L}[e^{at}f(t)] = F(s-a)$	$\mathcal{L}^{-1}[F(s-a)] = e^{at}f(t)$
$\mathcal{L}[f(t-a)\mathcal{U}(t-a)] = e^{-as}F(s)$	$\mathcal{L}^{-1}[e^{-as}F(s)] = f(t-a)\mathcal{U}(t-a)$