Building models with differential equations

Below are two examples of situations that can be modeled by differential equations: the spread of a contagious disease and the dissolution of brine in a mixing tank.

Example: A contagious disease (the flu, for example) is spread throughout a community by people coming into contact with other people. Let $x(t)$ denote the number of people who have contracted the disease by time t and $y(t)$ denote the number of people who have not as of time t. Assume that the rate at which the disease spreads is proportional to the number of encounters or interactions between these two populations. Further assume that the number of interactions is jointly proportional to the size of the populations x and y. Suppose that for some disease 5 people initially contracted it. Set up an IVP which models the spread of this disease through a community of n individuals.

Example: A vat contains 60**L** of water with 5 **kg** of salt dissolved in it. A salt water solution that contains 2 **kg** of salt per liter enters the vat at a rate of 3 **L/min**. Pure water is also flowing into the vat at a rate of 2 **L/min**. The solution in the vat is kept well mixed and is drained at a rate of 5 **L/min**, so that the rate in is the same as the rate out. Thus there is always 60**L** of brine at any given time. Set up an IVP which models the change in the amount of salt in the vat.