Essential Vocabulary

Below are a few definitions of terms we will use again and again throughout the course. You will need to be familiar with these.

Definition: (Ordinary Differential Equation) Any equation of the form

$$F(x, y, y', y'', \dots, y^{(m)}) = 0,$$

where $y^{(m)} = \frac{d^m y}{dx^m}$ is called an *ordinary differential equation* or ODE.

Definition: (Notations) We will interchange the Leibniz notation $\frac{d^k y}{dx^k}$ and the Newton notation $y^{(k)}$ for the k-th derivative of a function $y(x)$.

Definition: (Order) In a differential equation, the degree of the highest order derivative is called the *order* of the differential equation. That is,

$$F(x, y, y', \dots, y^{(n)}) = 0$$

is an n-th order differential equation of a differential equation of order n.

Definition: (Normal form) An n-th order differential equation is said to be in *normal form* if $y^{(n)}$ is solved in terms of all other terms. That is, the differential equation $F(x, y, y', y'', \dots, y^{(m)}) = 0$ is written in the form

$$\frac{d^n y}{dx^n} = f(x, y, y', \dots, y^{(n-1)}).$$

Definition: (Linearity) An n-th order differential equation is said to be *linear* if F is linear in $y, y', \dots, y^{(n)}$. This means that an n-th order differential equation when $F(x, y, y', \dots, y^{(n)}) = 0$ can be written in the form

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \dots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

where $g, a_0, a_1, \dots, a_n$ are functions of only x. If F can not be written this way, we call the differential equation *nonlinear*.

Definition: (Solution of an ODE) Any function φ, defined on an interval I and possessing at least n derivatives that are continuous on I, which when substituted into an n-th order ordinary differential equation reduces the equation to an identity, is said to be a *solution* of the differential equation on the interval.