Math 2410Q-003/014 — # Exam I — **Feb 19^{th}, 2018**

Name: ________________________________

Instructions:

- Answer each question to the best of your ability.
- All answers must be written clearly. Be sure to erase or cross out any work that you do not want graded. Partial credit can not be awarded unless there is legible work to assess.
- If you require extra space for any answer, you may use the back sides of the exam pages. Please indicate when you have done this so that I do not miss any of your work.

Academic Integrity Agreement

I certify that all work given in this examination is my own and that, to my knowledge, has not been used by anyone besides myself to their personal advantage. Further, I assert that this examination was taken in accordance with the academic integrity policies of the University of Connecticut.

Signed: ________________________________
(full name)

Questions:	1	2	3	4	5	6	Bonus	**Total**
Score:								

Percentage

1. (8 points) Assume k is a real valued constant. For which value(s) of k is $y = x^k$ a solution to the differential equation

$$x^2y'' + 4xy' - 4y = 0.$$

If no such value exists, explain why not.

2. (8 points) Consider the autonomous first order differential equation

$$\frac{dy}{dx} = y - y^3.$$

(a) (4 points) Find the critical points of this differential equation and sketch its phase portrait.

(b) (4 points) Sketch typical solution curves in the regions of the xy-plane determined by the graphs of the equilibrium solutions for this differential equation. (Make sure to include graphs of the equilibria themselves!)

3. (8 points) Find the general solution to the linear, first order differential equation

$$\frac{dy}{dx} + \tan(x)y = \sec(x)$$

on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Your solution should be presented as an explicit function $y(x)$.

Hint. $\displaystyle\int \tan(x)dx = \ln(\sec(x)) + c.$

4. (8 points) Determine a region of the xy-plane containing $(0,0)$ for which the differential equation

$$(4-y^2)y' = x^2$$

would have a unique solution whose graph passes through any point (x_0, y_0) in that region. Make sure to justify your answer.

5. (8 points) Consider the following initial value problem,

$$\frac{dy}{dx} = y(3 - xy), \quad y(0) = 1.$$

By hand, use Euler's method with $\Delta x = 1$ to approximate the value of $y(2)$. Use the table to record the necessary values of k, x_k, y_k, and $dy/dx = f(x_k, y_k)$ at each step. Justify your calculations if you wish to receive partial credit.

k	x_k	y_k	$f(x_k, y_k)$
0			
1			
2			

6. (8 points) Consider the differential equation

$$\frac{dy}{dx} = \sin(x+y) - \sin(x-y).$$

(a) (2 points) Verify that this DE has the trivial solution $y(x) = 0$.

(b) (6 points) Use $\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$ to solve this DE by separation of variables. Your solution may be presented as an implicit function.

Hint. $\displaystyle\int \csc(x)\, dx = -\ln(\cot(x) + \csc(x)) + c.$

7. (4 points) **Bonus:** Consider the following first order linear differential equation

$$\frac{dy}{dx} + P(x)y = f(x).$$

Show that if $y_1(x)$ and $y_2(x)$ are solutions to the above equation, then the difference of these functions

$$y_3(x) = y_1(x) - y_2(x)$$

is a solution to associated linear equation

$$\frac{dy}{dx} + P(x)y = 0$$

in which $f(x) = 0$.

8. (4 points) **Bonus:** In question 5, you were asked to approximate the solution to the IVP

$$\frac{dy}{dx} = y(3 - xy), \quad y(0) = 1$$

at $x = 2$. Find the explicit solution to this IVP.