Math 2210-002/010 Quiz 9 **Name:** Key **Due:** 4/15/19

This is a two-stage quiz. You will receive this back with each question graded pass/fail in our next class meeting. You have until the date specified above to submit corrections for partial credit.

1. (3 points) Let W be the set of all vectors of the form $\begin{bmatrix} s+3t \\ s-t \\ 2s-t \\ 4t \end{bmatrix}$. Show that W is a subspace of $\mathbb{R}^4$ by finding a spanning set for W.

$$W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ -1 \\ 4 \end{bmatrix} \right\}$$

If $\vec{x}$ in W then $\vec{x} = s\begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix} + t\begin{bmatrix} 3 \\ -1 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} s+3t \\ s-t \\ 2s-t \\ 4t \end{bmatrix}$

2. (3 points) If W is the set of all vectors of the form $\begin{bmatrix} a-2b \\ 3b+4 \\ 5a \end{bmatrix}$, is W a subspace of $\mathbb{R}^3$? Justify why or why not.

No, $\vec{0}$ not in W.

$\vec{0}$ in $\mathbb{R}^3$ is $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and if $a=b=0$,

$\begin{bmatrix} a-2b \\ 3b+4 \\ 5a \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix} \neq \vec{0}$. Every subspace must contain $\vec{0}$.

3. (4 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & -4 & -3 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

(i) (2 points) Is $\mathbf{u} = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ in Nul(A)? Justify your answer.

$$A\vec{u} = 3\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 0\begin{bmatrix} -4 \\ -2 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \vec{0}$$ so no.

$\vec{u}$ not in Nul(A).

(ii) (2 points) Give an explicit description of Nul(A) via a spanning set.

$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ in Nul(A) if $\vec{x}_1 = 4x_3 + 3x_4$

$x_2 = 2x_3 - x_4$

so $\vec{x} = x_3\begin{bmatrix} 4 \\ 2 \\ 1 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} 3 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

$$\Longrightarrow \text{Nul}(A) = \text{Span}\left\{\begin{bmatrix} 4 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 0 \\ 1 \end{bmatrix}\right\}.$$