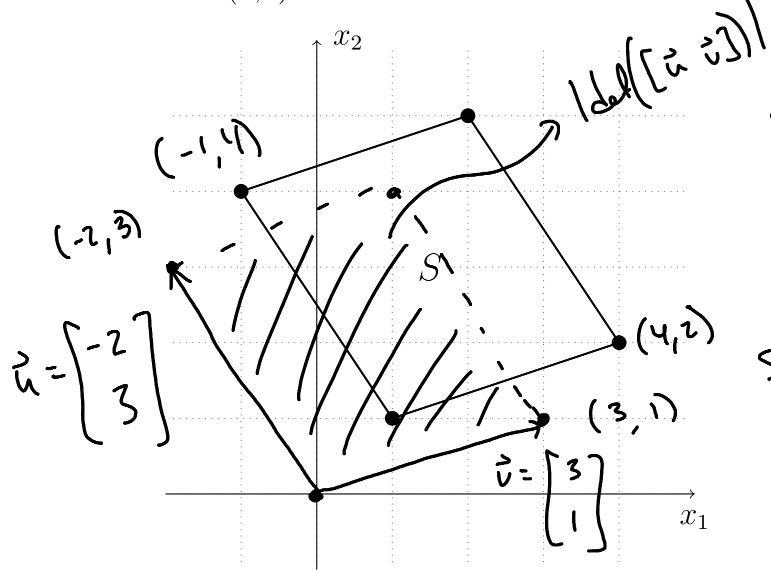
Math 2210-002/010 Quiz 8 **Name:** Key **Due:** 4/8/19

This is a two-stage quiz. You will receive this back with each question graded pass/fail in our next class meeting. You have until the date specified above to submit corrections for partial credit.

1. (5 points) Consider the parallelogram S plotted below with vertices (1,1), (-1,4), (2,5), and (4,2).



(i) (3 points) Find the area of S.

Shift S by $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$ to see spanned by $\vec{u} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

So area of S is

$$\left|\det \underbrace{\begin{bmatrix} -2 & 3 \\ 3 & 1 \end{bmatrix}}_{[\vec{u}\ \vec{v}]}\right| = |-11| \boxed{= 11}$$

(ii) (2 points) Define a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(\mathbf{x}) = A\mathbf{x}$ where

$$A = \begin{bmatrix} 1 & 4 \\ 1 & 2 \end{bmatrix}. \quad \leadsto \det A = -2$$

Compute the area of the parallelogram $T(S)$, the image of S under T.

$$\text{Area of } T(S) = |\det A| \cdot \text{"area of } S\text{"}$$

$$= |-2| \cdot 11$$

$$\boxed{= 22}$$

2. (5 points) Consider the vector space $\mathbb{P}_2$ (the space of polynomials of degree at most 2). Three vectors in this space are

$$p_1(t) = 1 + x$$
$$p_2(t) = 1 - x$$
$$p_3(t) = 3x^2 - 1.$$

Give 5 vectors from $\mathbb{P}_2$ that are elements of $\text{Span}\{p_1(t), p_2(t), p_3(t)\}$.

$p_1(t) = 1 + x$, $p_2(t) = 1 - x$, $p_3(t) = 3x^2 - 1$

$p_1(t) + p_2(t) = 2$ $\quad -3p_3(t) = -9x^2 + 3$

Recall that a vector is in the span of another set of vectors if it is a linear combination. So, any element of $\text{Span}\{p_1(t), p_2(t), p_3(t)\}$ is a linear combination of $p_1(t), p_2(t), p_3(t)$.

For example $2p_1(t) - p_2(t) + 2p_3(t)$

$$= 2(1+x) - (1-x) + 2(3x^2 - 1)$$
$$= 2 + 2x - 1 + x + 6x^2 - 2$$
$$\boxed{= 6x^2 + 3x - 1}$$