

This is a two-stage quiz. You will receive this back with each question graded pass/fail in our next class meeting. You have until the date specified above to submit corrections for partial credit.

1. (5 points) Find an LU factorization of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 3 & -2 \\ 1 & 3 & 0 & 0 \end{bmatrix}$$

$$A = LU$$

if

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 3 & -2 \\ 0 & 0 & 0 & -6 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 3 & -2 \\ 1 & 3 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 3 & -2 \\ 0 & 1 & -3 & -4 \end{bmatrix} \xrightarrow{R_3 + R_2} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 3 & -2 \\ 0 & 0 & 0 & -6 \end{bmatrix} = U$$

2. (5 points) Solve  $Ax = \vec{b}$  using the given LU factorization of A:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & -5 \\ 0 & -2 & 2 \\ 0 & 0 & 2 \end{bmatrix}.$$

$LU\vec{x} = \vec{b}$  so find  $\vec{y}$  s.t.  $L\vec{y} = \vec{b}$  and  $\vec{x}$  s.t.  $U\vec{x} = \vec{y}$ .

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ -1 & 1 & 0 & -4 \\ 2 & 0 & 1 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{bmatrix} \Rightarrow \vec{y} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$$

$$[L \vec{b}]$$

$$\begin{bmatrix} 4 & 3 & -5 & 2 \\ 0 & -2 & 2 & -2 \\ 0 & 0 & 2 & 2 \end{bmatrix} \sim \begin{bmatrix} 4 & 3 & -5 & 2 \\ 0 & -2 & 0 & -4 \\ 0 & 0 & 2 & 2 \end{bmatrix}$$

$$[U \vec{y}]$$

$$\sim \begin{bmatrix} 4 & 3 & 0 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 4 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \vec{x} = \begin{bmatrix} 1/4 \\ 2 \\ 1 \end{bmatrix}$$