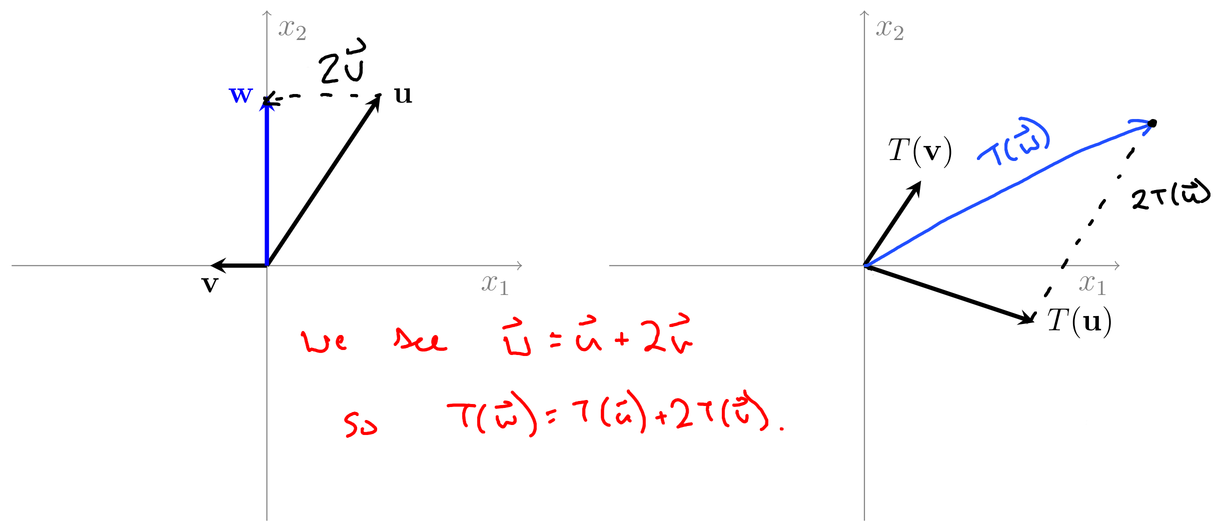
Math 2210-002/010 Quiz 5 **Name:** Key **Due:** 3/4/19

This is a two-stage quiz. You will receive this back with each question graded pass/fail in our next class meeting. You have until the date specified above to submit corrections for partial credit.

1. (3 points) The figure below shows vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ along with the images $T(\mathbf{u})$ and $T(\mathbf{v})$ under the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$. Carefully sketch and label the image $T(\mathbf{w})$ of $\mathbf{w}$ under T.



We see $\vec{w} = \vec{u} + 2\vec{v}$

so $T(\vec{w}) = T(\vec{u}) + 2T(\vec{v})$.

2. (3 points) Consider a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^2$ such that

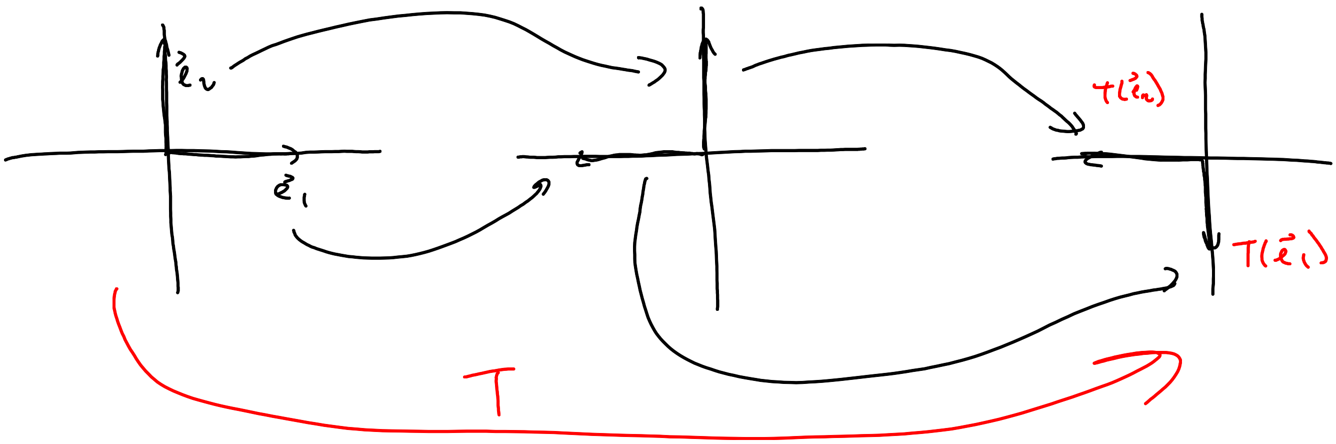
$$T\left(\begin{bmatrix} 1 \\ -2 \\ -3 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} -2 \\ 2 \\ 3 \\ -5 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 5 \end{bmatrix}.$$

Compute $T\left(\begin{bmatrix} -1 \\ 0 \\ 0 \\ -1 \end{bmatrix}\right) = T\left(\begin{bmatrix} 1 \\ -2 \\ -3 \\ 4 \end{bmatrix}\right) + T\left(\begin{bmatrix} -2 \\ 2 \\ 3 \\ -5 \end{bmatrix}\right)$

$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} -1 \\ 5 \end{bmatrix} = \boxed{\begin{bmatrix} 0 \\ 7 \end{bmatrix}}$$

By linearity: $\begin{bmatrix} -1 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -3 \\ 4 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \\ 3 \\ -5 \end{bmatrix}$

3. (4 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation which reflects a vector across the x_2 axis before rotating it counterclockwise by $\pi/2$ radians. Find the standard matrix of T.



So $T(\vec{e}_1) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$, $T(\vec{e}_2) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$$\Longrightarrow A = \left[T(\vec{e}_1) \;\; T(\vec{e}_2) \right] = \boxed{\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}}.$$