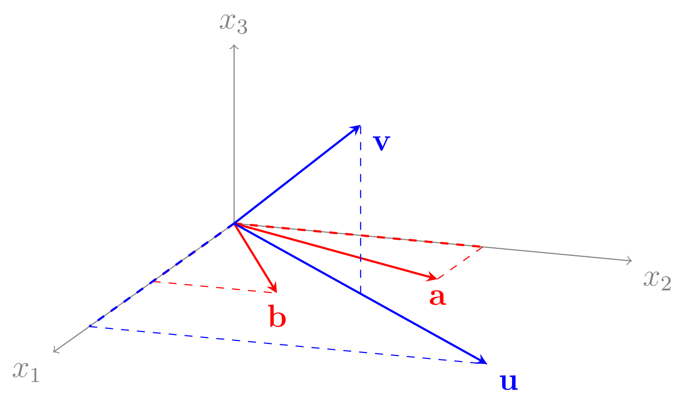
Math 2210-002/010 Quiz 4 **Name:** Key **Due:** 2/25/19

This is a two-stage quiz. You will receive this back with each question graded pass/fail in our next class meeting. You have until the date specified above to submit corrections for partial credit.

1. (2 points) Consider the vectors **a**, **b**, **u**, **v** given below.



$\vec{a}$ not in Span$\{\vec{b}\}$ so $\{\vec{a},\vec{b}\}$ is linearly independent.

Is $\{\mathbf{a}, \mathbf{b}, \mathbf{u}\}$ linearly independent? No. $\rightsquigarrow$ $\vec{u}$ in Span$\{\vec{a},\vec{b}\}$

Is $\{\mathbf{a}, \mathbf{b}, \mathbf{v}\}$ linearly independent? Yes. $\rightsquigarrow$ $\vec{v}$ not in Span$\{\vec{a},\vec{b}\}$

2. (4 points) Consider the vectors

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \mathbf{v_2} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix} \mathbf{v_3} = \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}.$$

Is $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

Notice $\vec{v}_1 + \vec{v}_3 = \vec{v}_2$, so no. $\rightsquigarrow$ $\vec{v}_1 - \vec{v}_2 + \vec{v}_3 = \vec{0}$ is a nontrivial linear dependence relation

—or—

$$\begin{bmatrix} 1 & 1 & 0 \\ -2 & 0 & 2 \\ 1 & 6 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 5 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

not pivot $\Rightarrow$ free variable

$[\vec{v}_1\ \vec{v}_2\ \vec{v}_3]\vec{x} = \vec{0}$ has nontrivial solution so $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent.

3. (4 points) Let $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$. Define a matrix transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ by

$$T(\mathbf{x}) = \mathbf{A}\mathbf{x} \text{ for each } \mathbf{x} \text{ in } \mathbb{R}^3.$$

If $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$, find the images $T(\mathbf{u})$ and $T(\mathbf{v})$ of $\mathbf{u}$ and v in $\mathbb{R}^2$.

$$T(\vec{u}) = A\vec{u} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \boxed{\begin{bmatrix} 9 \\ 12 \end{bmatrix}}.$$

$$T(\vec{v}) = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} - \begin{bmatrix} 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \boxed{\begin{bmatrix} 4 \\ 6 \end{bmatrix}}$$

Bonus: If $\mathbf{e_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{e_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{e_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, express $T(\mathbf{u})$ and $T(\mathbf{v})$ as linear combinations of $T(\mathbf{e_1})$, $T(\mathbf{e_2})$ and $T(\mathbf{e_3})$.

(4 points)

T is a matrix transformation so T is linear.

So, as $\vec{u} = \vec{e}_1 + \vec{e}_2 + \vec{e}_3$ and $\vec{v} = 2\vec{e}_1 - \vec{e}_2 + \vec{e}_3$

we have

$$\boxed{\begin{aligned} T(\vec{u}) &= T(\vec{e}_1) + T(\vec{e}_2) + T(\vec{e}_3) \\ \text{and } T(\vec{v}) &= 2T(\vec{e}_1) - T(\vec{e}_2) + T(\vec{e}_3). \end{aligned}}$$