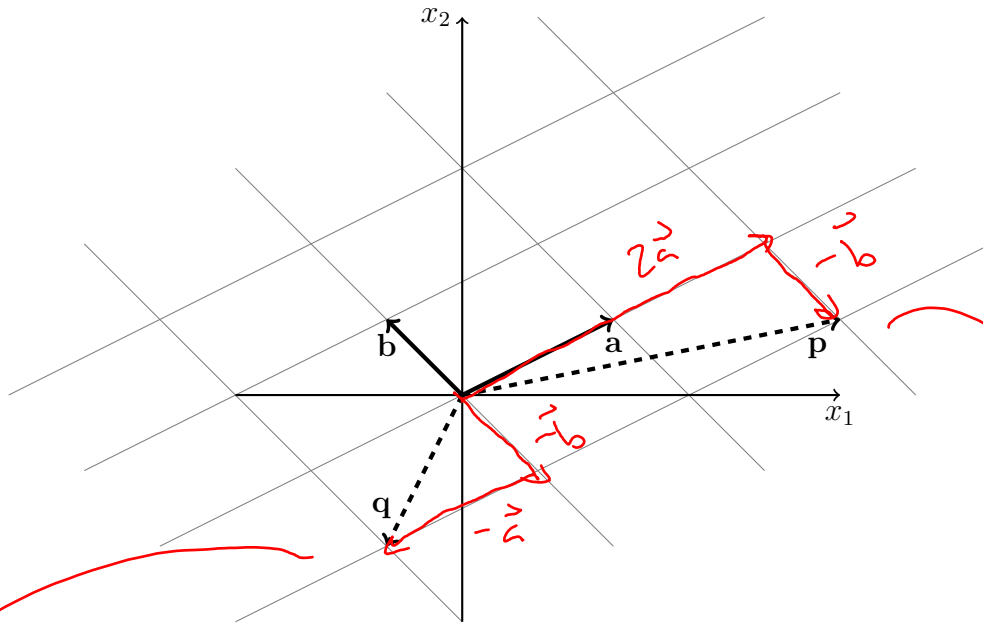


This is a two-stage quiz. You will receive this back with each question graded pass/fail in our next class meeting. You have until the date specified above to submit corrections for partial credit.

1. (4 points) The vectors  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{p}$  and  $\mathbf{q}$  from  $\mathbb{R}^2$  are graphed below. Note that  $\mathbf{p}$  and  $\mathbf{q}$  are in  $\text{Span}\{\mathbf{a}, \mathbf{b}\}$ .



- (i) (2 points) Based on the figure above, express  $\mathbf{p}$  as a linear combination of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\vec{p} = 2\vec{a} - \vec{b}$$

- (ii) (2 points) Based on the figure above, express  $\mathbf{q}$  as a linear combination of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\vec{q} = -\vec{a} - \vec{b}$$

2. (6 points) Consider the coefficient matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & -4 & 3 \\ -1 & -2 & 7 \end{bmatrix}$$

(i) (2 points) Give the reduced echelon form of  $A$ .

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & -4 & 3 \\ -1 & -2 & 7 \end{bmatrix} \xrightarrow[\sim]{\substack{R_2 + 2R_1 \\ R_3 + R_1}} \begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\xrightarrow[\sim]{\substack{R_3 - 6R_2 \\ R_1 + R_2}} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

← pivot  
← pivot  
← no pivot

Basic:  $x_1$        $x_3$   
Free:             $x_2$

(ii) (2 points) Let  $\mathbf{b}$  be any vector in  $\mathbb{R}^3$ . Does the equation  $A\mathbf{x} = \mathbf{b}$  necessarily have a solution? Justify your answer.

No, there is no pivot in the third row of  $A$ . (see §1.4)

(iii) (2 points) Is there a nontrivial solution to the equation  $A\mathbf{x} = \mathbf{0}$ ? Justify your answer.

Yes,  $x_2$  is a free variable.

Any nonzero choice of  $x_2$  yields a nontrivial solution. (see §1.5)