Math 2210-002/010 Quiz 2 **Name:** Key **Due:** 2/11/19

This is a two-stage quiz. You will receive this back with each question graded pass/fail in our next class meeting. You have until the date specified above to submit corrections for partial credit.

1. (6 points) Solve the following linear system by way of finding a particular matrix in reduced echelon form.

$$\begin{aligned} x_1 - 7x_2 \qquad + 6x_4 &= 5 \\ x_3 - 2x_4 &= -3 \\ -x_1 + 7x_2 - 4x_3 + 2x_4 &= 7 \end{aligned}$$

$$\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix} \overset{R3+R1}{\sim} \begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & -4 & 8 & 12 \end{bmatrix}$$

$$\overset{R3+4R2}{\sim} \begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

x_1 x_3 basic

x_2 x_4 free

Notice after clearing below the pivots (columns 1,3) we are done, the entries above all pivots are 0

$\underline{x_1} - 7x_2 + 6x_4 = 5$

$\underline{x_3} - 2x_4 = -3$

basic

parametric description of the solution set

$$\boxed{\begin{aligned} x_1 &= 5 + 7x_2 - 6x_4 \\ x_3 &= -3 + 2x_4 \\ x_2, x_4 &\text{ free} \end{aligned}}$$

2. Consider the vectors $\mathbf{a} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in $\mathbb{R}^2$.

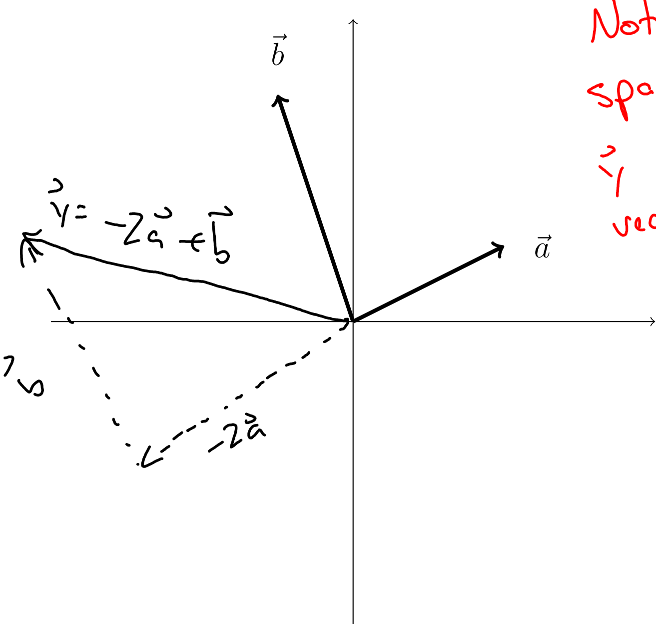
(i) (2 points) Compute the linear combination of $\mathbf{a}$ and $\mathbf{b}$ with weights $c_1 = -2$ and $c_2 = 1$.

$$\vec{y} = \underbrace{c_1\vec{a} + c_2\vec{b}}_{\text{the definition of a linear combination}} = (-2)\vec{a} + (1)\vec{b}$$

$$= -2\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -4-1 \\ -2+3 \end{bmatrix} = \boxed{\begin{bmatrix} -5 \\ 1 \end{bmatrix}}$$

(ii) (2 points) Sketch this linear combination below.



Notice $\vec{a}, \vec{b}$ span all of $\mathbb{R}^2$. $\vec{y}$ is only one vector in $\text{Span}\{\vec{a}, \vec{b}\}$