Math 2210-002/010 Quiz 10 **Name:** Key **Due:** 4/29/19

This is a two-stage quiz. You will receive this back with each question graded pass/fail in our next class meeting. You have until the date specified above to submit corrections for partial credit.

1. (4 points) Consider the matrix

$$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}.$$

(i) (2 points) Find the eigenvalues of A.

$$\det(A-\lambda I) = \begin{vmatrix} 4-\lambda & 2 \\ 3 & -1-\lambda \end{vmatrix} = (4-\lambda)(-1-\lambda) - 6$$
$$= \lambda^2 - 3\lambda - 4 - 6 = \lambda^2 - 3\lambda - 10$$
$$= (\lambda-5)(\lambda+2)$$

So $\det(A-\lambda I) = 0$

if $(\lambda-5)(\lambda+2) = 0$

$\Longrightarrow$ $\lambda_1 = 5$, $\lambda_2 = -2$ are the eigenvalues of A.

(ii) (2 points) Give an eigenvector for each eigenvalue you found in part (i).

$$[A-5I \;\; \vec{0}] \sim \begin{bmatrix} -1 & 2 & 0 \\ 3 & -6 & 0 \end{bmatrix} \sim \begin{bmatrix} -1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \vec{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

is e.vector w/ e.value $\lambda_1 = 5$.

$$[A+2I \;\; \vec{0}] \sim \begin{bmatrix} 6 & 2 & 0 \\ 3 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 3 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow v_2 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

is e.vector w/ e.value $\lambda_2 = -2$.

2. (6 points) Consider the matrix

$$A = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & -3 & 0 & 1 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}.$$

(i) (2 points) Give the characteristic polynomial of A.

$$\det(A - \lambda I) = (2-\lambda)(-3-\lambda)^2(-2-\lambda)$$

(ii) (2 points) Give the eigenvalues of A along with their multiplicites.

$\lambda_1 = 2$ w/ multiplicity 1

$\lambda_2 = -3$ w/ multiplicity 2

$\lambda_3 = -2$ w/ multiplicity 1

(iii) (2 points) Find a basis for the eigenspace of the least eigenvalue of A.

least e.value is $\lambda_2 = -3$ so

$$[A+3I \ \ \vec{0}] = \begin{bmatrix} 5 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 5 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$\Rightarrow$ $5x_1 = x_3$

x_2 free

x_3 free

$x_4 = 0$

$$\Rightarrow \vec{x} = x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 0 \\ 5 \\ 0 \end{bmatrix}$$

So espace of λ_2 is $\text{Span}\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 5 \\ 0 \end{bmatrix} \right\}$

$\Rightarrow$ basis is $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 5 \\ 0 \end{bmatrix} \right\}$.