Math 2210-006/011 Quiz 9 **Name:** Key **Due:** 11/11/19

This is a two-stage quiz. You will receive this back with each question graded pass/fail in our next class meeting. You have until the date specified above to submit corrections for partial credit.

1. (3 points) Let W be the set of all vectors of the form $\begin{bmatrix} s+3t \\ s-t \\ 2s-t \\ 4t \end{bmatrix}$. Show that W is a subspace of $\mathbb{R}^4$ by finding a spanning set for W.

$$W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ -1 \\ 4 \end{bmatrix} \right\}$$

as all $\vec{u}$ in W satisfy

$$\vec{u} = s\begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix} + t\begin{bmatrix} 3 \\ -1 \\ -1 \\ 4 \end{bmatrix} \text{ for some } s, t.$$

2. (3 points) If W is the set of all vectors of the form $\begin{bmatrix} a-2b \\ 3b+4 \\ 5a \end{bmatrix}$, is W a subspace of $\mathbb{R}^3$? Justify why or why not.

No. $\vec{0}$ of $\mathbb{R}^3$, $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, is not in W.

For any choice of a, b

$$\begin{bmatrix} a-2b \\ 3b+4 \\ 5a \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

3. (4 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & -4 & -3 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

(i) Is $\mathbf{u} = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ in Nul(A)? Justify your answer.

$$A\vec{u} = 3\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 0\begin{bmatrix} -4 \\ -2 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\hookrightarrow$ so no, $\vec{u}$ <u>not</u> in Nul(A).

(ii) Give an explicit description of Nul(A) via a spanning set.

$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ in Nul(A) if $A\vec{x} = \vec{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, and

$$\left[\begin{array}{cccc|c} 1 & 0 & -4 & -3 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{aligned} x_1 &= 4x_3 + 3x_4 \\ x_2 &= 2x_3 - x_4 . \end{aligned}$$

So $\vec{x}$ in Nul(A) if $\vec{x} = x_3\begin{bmatrix} 4 \\ 2 \\ 1 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} 3 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

Thus $\text{Nul}(A) = \text{Span}\left\{ \begin{bmatrix} 4 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}.$