Math 2210-006/011 Quiz 8 **Name:** Key **Due:** 11/4/19

This is a two-stage quiz. You will receive this back with each question graded pass/fail in our next class meeting. You have until the date specified above to submit corrections for partial credit.

1. (5 points) Recall that $M_{2\times 2}$ is the vector space of 2×2 matrices with real entries. Verify that the addition of this vector space (the usual addition of matrices) satisfies axiom 3 of the 10 vector space axioms:

$$\text{For all } \mathbf{u}, \mathbf{v}, \mathbf{w} \text{ in } V \text{ we have } (\mathbf{u}+\mathbf{v})+\mathbf{w} = \mathbf{u}+(\mathbf{v}+\mathbf{w})$$

Take $\vec{u}, \vec{v}, \vec{w}$ arbitrary in $M_{2\times 2}$:

$$\vec{u} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix}$$

$$(\vec{u}+\vec{v})+\vec{w} = \left(\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} + \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \right) + \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix}$$

$$= \begin{bmatrix} a_1+b_1 & a_2+b_2 \\ a_3+b_3 & a_4+b_4 \end{bmatrix} + \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix}$$

$$= \begin{bmatrix} (a_1+b_1)+c_1 & (a_2+b_2)+c_2 \\ (a_3+b_3)+c_3 & (a_4+b_4)+c_4 \end{bmatrix}$$

$$= \begin{bmatrix} a_1+(b_1+c_1) & a_2+(b_2+c_2) \\ a_3+(b_3+c_3) & a_4+(b_4+c_4) \end{bmatrix}$$

$$= \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} + \begin{bmatrix} b_1+c_1 & b_2+c_2 \\ b_3+c_3 & b_4+c_4 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} + \left(\begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} + \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix} \right) = \vec{u}+(\vec{v}+\vec{w}).$$

✓

2. (5 points) Consider the vector space $\mathbb{P}_3$ (the space of polynomials of degree at most 3). Three vectors in this space are

$$p_1(t) = 1 + t$$
$$p_2(t) = 1 - t$$
$$p_3(t) = 3t^2 - 1.$$

Give 5 vectors from $\mathbb{P}_3$ that are elements of Span$\{p_1, p_2, p_3\}$.

$p_1(t) = 1 + t$

$p_2(t) = 1 - t$

$p_3(t) = 3t^2 - 1$

$-p_1(t) = -1 - t$

$-p_2(t) = -1 + t$

$-p_3(t) = -3t^2 + 1$

$(p_1 + p_2)(t) = 2$

$(p_1 + p_3)(t) = 3t^2 - t$

$(2p_2 - p_3)(t) = 3 - 2t - 3t^2$

$(p_1 + p_2 + p_3)(t) = 3t^2 + 1$

Any 5 of these vectors is suitable.

(Many more possible answers.)