This is a two-stage quiz. You will receive this back with each question graded pass/fail in our next class meeting. You have until the date specified above to submit corrections for partial credit.

1. (5 points) Compute the determinant of

$$A = \begin{bmatrix} 4 & 1 & 2 \\ 4 & 0 & 3 \\ 3 & -2 & 5 \end{bmatrix}.$$

$$\det A = \sum_{i=1}^{3} a_{i2} C_{i2}$$

$$= a_{12} C_{12} + a_{22} C_{22} + a_{32} C_{32}$$

$$= 1(-1)\begin{vmatrix} 4 & 3 \\ 3 & 5 \end{vmatrix} + 0 + (-2)(-1)\begin{vmatrix} 4 & 2 \\ 4 & 3 \end{vmatrix}$$

$$= -11 + 2(4) \boxed{= -3}$$

2. (5 points) Let H be an $n \times n$ matrix and $\mathbf{x}$ in $\mathbb{R}^n$. Suppose for a fixed $\mathbf{c}$ in $\mathbb{R}^n$, the equation $H\mathbf{x} = \mathbf{c}$ is inconsistent.

(i) Is H invertible? Justify your answer.

No. If H is invertible, $H\vec{x} = \vec{c}$ is consistent for all $\vec{c}$ in $\mathbb{R}^n$ by IMT.

(ii) Does the homogeneous equation $H\mathbf{x} = \mathbf{0}$ have a nontrivial solution? Justify your answer.

Yes. Since H not invertible, IMT guarentees $H\vec{x} = \vec{0}$ does not have only the trivial solution, so it must have a non-trivial solution.