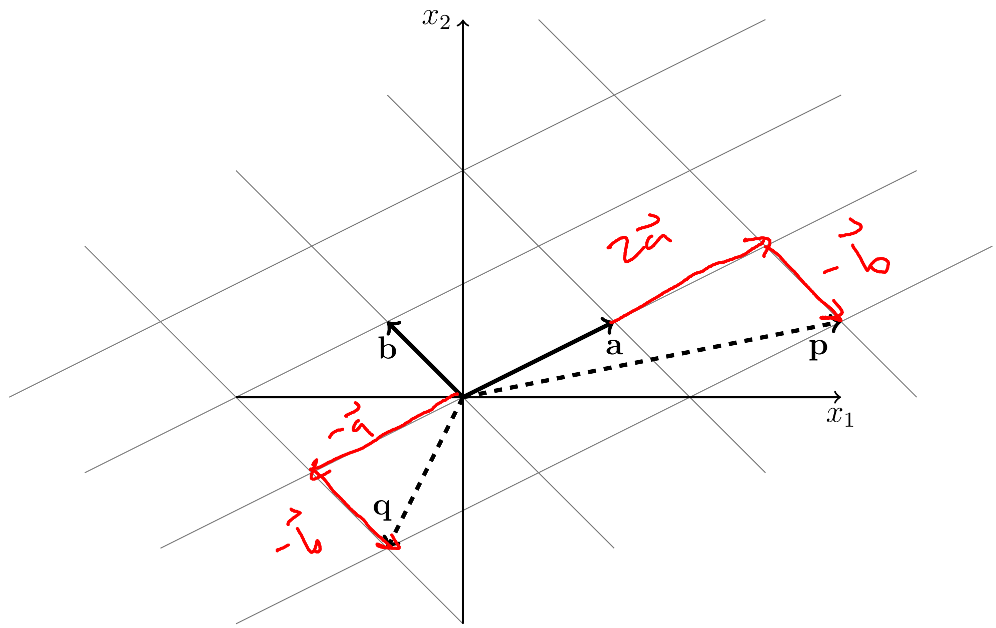
Math 2210-002/010 Quiz 4 **Name:** Key 9/18/19

This quiz will be graded with partial credit.

1. (4 points) The vectors **a**, **b**, **p** and **q** from $\mathbb{R}^2$ are graphed below. Note that **p** and **q** are in Span$\{\mathbf{a}, \mathbf{b}\}$.



(i) (2 points) Based on the figure above, express **p** as a linear combination of **a** and **b**.

$$\vec{p} = 2\vec{a} - \vec{b}$$

(ii) (2 points) Based on the figure above, express **q** as a linear combination of **a** and **b**.

$$\vec{q} = -\vec{a} - \vec{b}$$

2. (6 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & -4 & 3 \\ -1 & -2 & 7 \end{bmatrix}$$

Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 2 \\ -4 \\ -2 \end{bmatrix}$ and $\mathbf{a}_3 = \begin{bmatrix} -1 \\ 3 \\ 7 \end{bmatrix}$ be the columns of A.

(i) (4 points) Is $\mathbf{b}_1 = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}$ a linear combination of the columns of A? If so, give weights x_1, x_2 and x_3 that witness this. If not, justify why.

$$[A\ \vec{b}] = \begin{bmatrix} 1 & 2 & -1 & 1 \\ -2 & -4 & 3 & -1 \\ -1 & -2 & 7 & 5 \end{bmatrix} \underset{R3:R3+R1}{\overset{R2:R2+2R1}{\sim}} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 6 & 6 \end{bmatrix}$$

$$\underset{R1:R1+R2}{\overset{R3:R3-6R2}{\sim}} \begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{array}{l} x_1 = 2 - 2x_2 \\ x_3 = 1 \\ x_2 \text{ free.} \end{array}$$

$$\boxed{x_2 = 0 \Rightarrow x_1 = 2, x_3 = 1}$$

$$\hookrightarrow 2\vec{a}_1 + 0\vec{a}_2 + \vec{a}_3 = \begin{bmatrix} 2 \\ -4 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} = \vec{b}. \checkmark$$

(ii) (2 points) Let $\mathbf{b}$ be any vector in $\mathbb{R}^3$. Does the equation $A\mathbf{x} = \mathbf{b}$ necessarily have a solution? Justify your answer.

No, A would need a pivot in every row for this to be true.