Math 2210-~~002/010~~ 006/019 Quiz 11 **Name:** Key

This quiz will be graded with partial credit.

1. (10 points) If possible, diagonalize

$$A = \begin{bmatrix} -2 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & -2 & -2 \end{bmatrix}.$$

That is find matices P and D such that $A = PDP^{-1}$ with D diagonal.

Eigenvalues: $\det(A - \lambda I) = \begin{vmatrix} -2-\lambda & 2 & 0 \\ 0 & -1-\lambda & 0 \\ 0 & -2 & -2-\lambda \end{vmatrix} = (-2-\lambda)\begin{vmatrix} -1-\lambda & 0 \\ -2 & -2-\lambda \end{vmatrix}$

$= (-2-\lambda)^2(-1-\lambda)$

$\Rightarrow \lambda_1 = -2, \quad \lambda_2 = -1$

Eigenvectors: $[A - \lambda_1 I \;\; \vec{0}] = [A + 2I \;\; \vec{0}] = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

So,

2 linearly independent e.vectors (for λ_1) $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

$\Rightarrow x_1, x_3$ free, $x_2 = 0$

$[A - \lambda_2 I \;\; \vec{0}] = [A + I \;\; \vec{0}] = \begin{bmatrix} -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

So $\vec{v}_3 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$ is an e.vector for λ_2.

$\Rightarrow x_1 = x_3, \; 2x_2 = -x_3$, x_3 free

$\Rightarrow P = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}, \; D = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ and

$A = PDP^{-1}$