Math 2210-006/011 Quiz 10 **Name:** Key **Due:** 12/2/19

This is a two-stage quiz. You will receive this back with each question graded pass/fail in our next class meeting. You have until the date specified above to submit corrections for partial credit.

1. (4 points) Consider the matrix

$$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}.$$

(i) Find the eigenvalues of A.

$$\det(A - \lambda I) = \begin{vmatrix} 4-\lambda & 2 \\ 3 & -1-\lambda \end{vmatrix} = (4-\lambda)(-1-\lambda) - 6$$

$$= \lambda^2 - 3\lambda - 4 - 6 = \lambda^2 - 3\lambda - 10$$

$$= (\lambda - 5)(\lambda + 2)$$

So $\det(A - \lambda I) = 0$ if $(\lambda - 5)(\lambda + 2) = 0$

so $\lambda_1 = 5$ and $\lambda_2 = -2$

are the eigenvalues of A.

(ii) Give an eigenvector for each eigenvalue you found in part (i).

$$\begin{bmatrix} A - 5I & \vec{0} \end{bmatrix} \sim \begin{bmatrix} -1 & 2 & 0 \\ 3 & -6 & 0 \end{bmatrix} \sim \begin{bmatrix} -1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{matrix} (A - 5I)\vec{x} = \vec{0} \\ \text{if } \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix}. \end{matrix}$$

If $\vec{x}$ is not $\vec{0}$, it is an eigenvector w/ eigenvalue 5

in particular, $\boxed{\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}}$.

Simillarly,

$$\begin{bmatrix} A + 2I & \vec{0} \end{bmatrix} \sim \begin{bmatrix} 6 & 2 & 0 \\ 3 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 3 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Longrightarrow \boxed{\vec{v}_2 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}}$$

is an eigenvector with eigenvalue -2.

2. (6 points) Consider the matrix

$$A = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & -3 & 0 & 1 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}.$$

(i) Give the characteristic polynomial of A.

$$\det(A - \lambda I) = (2-\lambda)(-3-\lambda)^2(-2-\lambda)$$

(ii) Give the eigenvalues of A along with their multiplicites.

$\lambda_1 = 2$ 1

$\lambda_2 = -3$ with multiplicity 2

$\lambda_3 = -2$ 1

(iii) Find a basis for the eigenspace of the least eigenvalue of A.

Least eigenvalue is $\lambda_2 = -3$. Need basis for $Nul(A+3I)$.

$$[A+3I \;\; \vec{0}] = \begin{bmatrix} 5 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 5 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Longrightarrow \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ is in $Nul(A+3I)$ if $x_1 = \frac{1}{5}x_3$, $x_4 = 0$ with x_2, x_3 free.

So $\vec{x} = x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 1/5 \\ 0 \\ 1 \\ 0 \end{bmatrix}$. $\Longrightarrow Nul(A+3I) = \text{Span}\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/5 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$.

Thus a basis for the eigenspace of $\lambda_2 = -3$ $(Nul(A+3I))$ is $\boxed{\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/5 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}}$