1.2 Row Reduction and echelon forms

Friday, January 25, 2019 1:00 AM

Recap last time: three linear systems, three graphs
three parts of fact, consistent def

Today we will discuss an algorithm, row reduction, that will allow us to determine if any linear system (regardless of size) is consistent and what it's solution set is.

Let's motivate it with a single system.

Ex) Solve the following system:

$$\frac{1}{2}x - y = 2 \quad E1$$

$$\frac{1}{2}x + y = 4 \quad E2$$

Recall the three operations one [illegible] do systems of equations

Recall the three ... we
can do on systems of equations
with modifying what they state.

1) Reorder, 2) Scale, 3) Add

Notice

$$\begin{aligned} \tfrac{1}{2}x_1 - x_2 &= 2 \\ \tfrac{1}{2}x_1 + x_2 &= 4 \end{aligned} \xrightarrow{E1+E2} \begin{aligned} x_1 \quad &= 6 \\ \tfrac{1}{2}x_1 + x_2 &= 4 \end{aligned}$$

$$\xrightarrow{\frac{1}{2}E1} \begin{aligned} \tfrac{1}{2}x_1 \quad &= 3 \\ \tfrac{1}{2}x_1 + x_2 &= 4 \end{aligned}$$

$$\xrightarrow{E2-E1} \begin{aligned} \tfrac{1}{2}x \quad &= 3 \\ x_2 &= 1 \end{aligned}$$

So $x=6$, $x_2=1$ is the solution!

<u>Associated matrix</u> give vocab handout

Notice I can summarize the information about a system using a matrix about a system using

$$\begin{aligned} \tfrac{1}{2}x_1 - x_2 &= 2 \\ \tfrac{1}{2}x_1 + x_2 &= 4 \end{aligned} \Rightarrow \begin{pmatrix} \frac{1}{2} & -1 \\ \frac{1}{2} & 1 \end{pmatrix}$$ coefficient matrix

$$\longrightarrow \left(\begin{array}{cc|c} \frac{1}{2} & -1 & 2 \\ \frac{1}{2} & 1 & 4 \end{array}\right)$$ augmented matrix

Instead of working with the equations, we manipulate the rows of the augmented matrix using elementary row operations:

$$\begin{array}{c} R1 \\ R2 \end{array} \begin{pmatrix} \frac{1}{2} & -1 & 2 \\ \frac{1}{2} & 1 & 4 \end{pmatrix} \xrightarrow{R_1 + R_2} \begin{pmatrix} 1 & 0 & 6 \\ \frac{1}{2} & 1 & 4 \end{pmatrix}$$

$$\xrightarrow{R_2 - \frac{1}{2}R_1} \begin{pmatrix} 1 & 0 & 6 \\ 0 & 1 & 1 \end{pmatrix}$$

Translating into an equivalent system gives

$x_1 = 6$

$x_2 = 1$.

Generalizing this process yields the row reduction algorithm, but first, a few definitions.

Read through vocab, and handout part 2. Go through it.

Hand out part 3, and go over it.