Math 2110-003 Worksheet 16.8 and 16.9 **Name:** ______________________ **Not due.**

The divergence theorem and Stokes' theorem

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1. Let $\mathbf{F} = \langle z, y, x\rangle$ and let S be the surface $x^2 + y^2 + z^2 = 16$ with outward orientation.
   (a) Compute the flux of $\mathbf{F}$ across the surface S using the **definition** of a surface integral.
   Hint: You may use the parametrization $\mathbf{r}(\phi, \theta) = \langle 4\sin\phi\cos\theta, 4\sin\phi\sin\theta, 4\cos\phi\rangle$ but make sure you understand where this comes from and could reproduce it on your own if necessary. Here
   $$\mathbf{r}_\phi \times \mathbf{r}_\theta = \langle 16\sin^2\phi\cos\theta, 16\sin^2\phi\sin\theta, 16\sin\phi\cos\phi\rangle.$$
   (b) Use the Divergence Theorem to find the flux, and make sure your answer agrees with part (a).

2. Let S be the surface of the solid bounded by $y^2 + z^2 = 1$, $x = -1$ and $x = 2$ and let $\mathbf{F} = \langle 3xy^2, xe^z, z^3\rangle$. Calculate the flux of $\mathbf{F}$ across the surface S, assuming it has positive orientation.

3. Let S be the surface $x^2 + y^2 + z^2 = 4$ with positive orientation and let $\mathbf{F} = \langle x^3 + y^3, y^3 + z^3, z^3 + x^3\rangle$. Evaluate the surface integral
   $$\iint_S \mathbf{F} \bullet d\mathbf{S}.$$

4. Let $\mathbf{F} = \langle P, Q, R\rangle$ be a vector field and S and a closed surface with positive orientation containing a solid E. Use the Divergence Theorem to compute
   $$\iint_S \operatorname{curl} F \bullet d\mathbf{S}.$$

5. **Optional:** Compute $\displaystyle\iint_S \mathbf{F} \bullet d\mathbf{S}$ if $\mathbf{F} = \langle x^2y, -xy^2, 4z(z^2 - 1)\rangle$ and S is the unit cube with outward orientation with vertices at $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, $(1,1,0)$, $(0,0,1)$, $(1,0,1)$, $(0,1,1)$, and $(1,1,1)$, minus the top face. (So S is a box without a lid.)
   Notice that S is not closed. We can still use the Divergence Theorem to save us some work. How might we do this? *Hint:* If we compute the integral directly, we need to do five constituent integrals (one for each side). If we use the Divergence Theorem we will only need to compute *two* integrals to compute $\iint_S \mathbf{F} \bullet d\mathbf{S}$.

6. Let $\mathbf{F} = \langle x^2 \sin z, y^2, xy\rangle$ and S be the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the xy-plane with an orientation such the $\mathbf{n}$ at the vertex points in the direction of the positive z-axis. Use Stokes' Theorem to compute
   $$\iint_S \operatorname{curl} \mathbf{F} \bullet d\mathbf{S}.$$

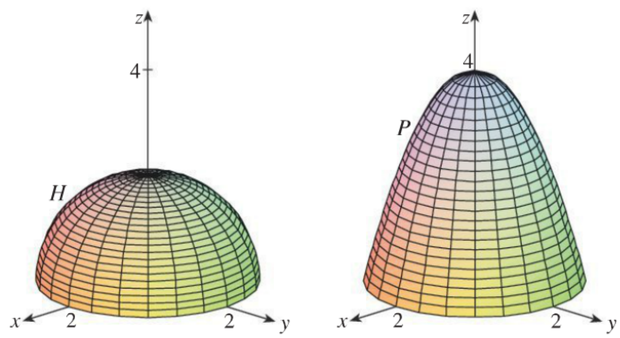
7. Let $\mathbf{F} = \langle \arctan(x^2yz^2, x^2y, x^2z^2 \rangle$ and S be the cone $x = \sqrt{y^2+z^2}$, $0 \leqslant x \leqslant 2$, oriented in the direction of the positive x-axis. Compute

$$\iint_S \operatorname{curl} \mathbf{F} \bullet d\mathbf{S}.$$

8. Let $\mathbf{F} = \langle 2y, xz, x+y \rangle$ and C is the curve of intersection of the plane $z = y+2$ and the cylinder $x^2+y^2 = 1$.

9. A hemisphere H and a portion P of a paraboloid are shown below. Suppose $\mathbf{F}$ is a vector field on $\mathbb{R}^3$ whose components have continuous partial derivatives. Explain why

$$\iint_H \operatorname{curl} \mathbf{F} \bullet d = \iint_P \operatorname{curl} \mathbf{F} \bullet d\mathbf{S}.$$



10. **Optional:** Suppose S is a surface with boundary curve C that satisfy the hypotheses of Stokes' Theorem and f, and g have continuous second-order partial derivatives. Use problem 3b from worksheet 16.5 to show

   (a) $\int_C (f\nabla g) \bullet d\mathbf{r} = \iint_S (\nabla f \times \nabla g) \bullet d\mathbf{S}$

   (b) $\int_C (f\nabla f) \bullet d\mathbf{r} = 0$