Math 2110-003 Worksheet 16.7 **Name:** ____________________________ **Due:** 12/6/2017

# Surface integrals

1. Evaluate the surface integral $\iint_S x\,dS$ if $S$ is the helicoid $\mathbf{r}(u,v) = \langle u\cos v, u\sin v, v\rangle$ with $(u,v) \in [0,1]\times[0,\pi]$.

2. Evaluate the surface integral $\iint_S 2\,dS$ if $S$ is the portion of the plane $2x+3y+z=6$ in the first octant.

3. Evaluate the surface integral $\iint_S x^2z^2\,dS$ if $S$ is the portion of the cone $z=\sqrt{x^2+y^2}$ with $1 \leqslant z \leqslant 4$.

4. Give two orientations of the surface $S$ given by the graph of $z=g(x,y)$ for $(x,y)\in D$. (*Hint:* Parametrize the surface by $r(x,y) = \langle x, y, g(x,y)\rangle$ with $(x,y)\in D$.)

5. Let $\mathbf{F} = \langle e^{yz}, xze^{yz}, xye^{yz}\rangle$.

   (a) If $S$ is the portion of the $xz$-plane with $-1 \leqslant x \leqslant 1$ and $0 \leqslant z \leqslant 1$ oriented in the direction of the positive $y$-axis, evaluate the surface integral $\iint_S \mathbf{F}\bullet d\mathbf{S}$.

   (b) Show that $\mathbf{F}$ is a conservative vector field.

   (c) If $S$ is the surface $z=\sqrt{x^2+y^2}$ with $0 \leqslant z \leqslant 4$ and outward orientation, evaluate the surface integral $\iint_S \operatorname{curl}\mathbf{F}\bullet d\mathbf{S}$.

   (d) If $C$ is the curve of intersection between the cylinder $x^2+y^2=4$ and the plane $3x-2y+7z=12$. (What is the shape of this intersection?) Evaluate the line integral $\int_C \mathbf{F}\bullet d\mathbf{r}$.

6. Let $\mathbf{F} = \langle -y, z, -x\rangle$.

   (a) If $S$ is the surface $x^2+y^2 \leqslant 4$, $z=0$, with upward orientation, evaluate $\iint_S \mathbf{F}\bullet d\mathbf{S}$.

   (b) Compute $\operatorname{curl}\mathbf{F}$.

   (c) If $S$ is the portion of the surface $z=4-x^2-y^2$ with $z \geqslant 0$ and upward orientation, evaluate the surface integral $\iint_S \operatorname{curl}\mathbf{F}\bullet d\mathbf{S}$.