Math 2110-003 Worksheet 16.5 **Name:** ____________________ **Due:** 11/22/2017

Curl and divergence

1. For each of the following, either compute the expression or explain why it doesn't make sense (i.e. is not defined). Assume that $f(x, y, z) = x^2y + xz - 1$ and $\mathbf{F} = \langle z, x, y \rangle$.

   For example, we can say div f does not make sense as div is an operation defined on vector fields, not scalar functions.

   (a) $\operatorname{div}(\operatorname{curl} f)$
   (b) $\operatorname{div}(\operatorname{curl} \mathbf{F})$
   (c) $\operatorname{curl}(\operatorname{div} \mathbf{F})$
   (d) $\operatorname{curl}(\operatorname{div} \mathbf{f})$
   (e) $\operatorname{div}(\operatorname{div} \mathbf{F})$
   (f) $\nabla f \bullet \mathbf{F}$
   (g) $\operatorname{curl}(\nabla f)$
   (h) $\operatorname{curl} \mathbf{F} + \operatorname{div} \mathbf{F}$
   (i) $\operatorname{div}(\nabla f + \mathbf{F})$

2. Using the same scalar function $f(x, y, z)$ and vector field $\mathbf{F} = \langle z, x, y \rangle$ given above, evaluate each of the following expressions. You may use the fundamental theorem for line integrals or Green's theorem if they apply.

   Hint: They will in all but one case.

   (a) $\int_{C_1} \operatorname{curl} \mathbf{F} \bullet d\mathbf{r}$ where C_1 is the line segment from $(-1, 3, 5)$ to $(3, -1, -2)$.
   (b) $\int_{C_2} \nabla f \bullet d\mathbf{r}$ where C_2 is the portion of the parabola given by $\mathbf{r}(t) = \langle t^2, t, 3t \rangle$ with $-1 \leqslant t \leqslant 1$.
   (c) $\int_{C_3} \operatorname{div} \mathbf{F} ds$ where C_3 is the curve given by $\mathbf{r}(t) = \langle e^{t^2}, \ln(t^3 + 1), 1 \rangle$ with $t \in [0, 5]$.
   (d) $\int_{C_4} \operatorname{curl} \mathbf{F} \bullet d\mathbf{r}$ where C_4 is the ellipse given by $\mathbf{r}(t) = \langle \cos t, \sin t, \cos t \rangle$ with $t \in [0, 2\pi]$.

3. **Optional:** Assuming the appropriate partial derivatives exist and are continuous for a scalar function f and vector field $\mathbf{F} = \langle P, Q, R \rangle$ show that div and curl satisfy familiar rules of differentiation.

   (a) Prove $\operatorname{div}(f\mathbf{F}) = f \operatorname{div} \mathbf{F} + F \bullet \nabla f$.
   (b) Prove $\operatorname{curl}(f\mathbf{F}) = f \operatorname{curl} \mathbf{F} + (\nabla f) \times \mathbf{F}$.

4. **Optional:** Assuming the continuity of the appropriate partial derivatives, we have seen that all vector fields of the form $\mathbf{F} = \nabla g$ satisfy the equation $\operatorname{curl} F = 0$ and that vector fiels of the form $\mathbf{F} = \operatorname{curl} G$ satisfy the equation $\operatorname{div} F = 0$. This suggests the question:

   are there any equations that all functions of the form $f = \operatorname{div} \mathbf{G}$ must satisfy?

   Show the answer to this question is "No." by proving that *every* continuous function on $\mathbb{R}^3$ is the divergence of some vector field.

   Hint: If $g(x, y, z) = \int_0^x f(t, y, z)\, dt$ then what is $\dfrac{\partial g}{\partial x}$?