Math 2110-003 Worksheet 16.4 **Name:** ____________________________ **Due:** 11/15/2017

Green's Theorem

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1. Compute $\int_C y^2\,dx + 3xy\,dy$ where C is the boundary of the semiannular region D in the upper half-plane between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

2. Compute $\int_C \mathbf{F} \bullet d\mathbf{r}$ if $\mathbf{F} = \langle y\cos x - xy\sin x, xy + x\cos x\rangle$ and C is the triangle formed by going from $(0,0)$ to $(2,4)$ to $(2,0)$ and back to $(0,0)$.

3. Suppose $\mathbf{F}$ is a conservative vector field. Use Green's theorem to prove that $\int_C \mathbf{F} \bullet d\mathbf{r} = 0$ for any closed curve C (that is smooth and does not self-intersect).

   Hint: Recall that if $\mathbf{F} = \langle P, Q\rangle$ then we know how $\frac{\partial P}{\partial y}$ and $\frac{\partial Q}{\partial x}$ relate.

4. Let $\mathbf{F}(x,y) = \langle y^2, x^2y\rangle$ and C is the rectangle with vertices $(0,0)$, $(5,0)$, $(5,4)$, and $(0,4)$ and positive orientation. Consider the line integral $\int_C \mathbf{F} \bullet d\mathbf{r}$.

   (a) Compute the line integral directly.

   (b) Compute this line integral using Green's theorem.