Math 2110-003 Worksheet 16.3 **Name:** ____________________ **Due:** 11/8/2017

The fundamental theorem for line integrals

1. Let $f(x, y) = 3x + x^2y^2$ and C be the arc of the hyperbola $y = 1/x$ from $(1, 1)$ to $(4, 1/4)$. Compute $\int_C \nabla f \bullet d\mathbf{r}$.

2. (a) Is $\mathbf{F}(x, y) = \langle xy + y^2, x^2 + 2xy\rangle$ conservative? Justify your answer and if so, provide a potential function f for $\mathbf{F}$, that is, f such that $\mathbf{F} = \nabla f$.

   (b) Compute $\int_C \mathbf{F} \bullet d\mathbf{r}$ where C is the line segment from $(0, 0)$ to $(1, 1)$.

3. (a) Is $\mathbf{F}(x, y) = \langle x^2y^3, x^3y^2\rangle$ conservative? Justify your answer and if so, provide a potential function f for $\mathbf{F}$, that is, f such that $\mathbf{F} = \nabla f$.

   (b) Compute $\int_C \mathbf{F} \bullet d\mathbf{r}$ where C is given by $\mathbf{r}(t) = \langle t^3 - 2t, t^3 + 2t\rangle$ for $t \in [0, 1]$

4. Let C be the curve which starts at $(1/2, 0)$ and traces the ellipse $4x^2 + 9y^2 = 1$ in the counterclockwise direction, only once. If $\mathbf{F}(x, y) = \langle y^2e^{xy}, (1 + xy)e^{xy}\rangle$, calculate $\int_C \mathbf{F} \bullet d\mathbf{r}$.

5. Let $\mathbf{F}(x, y, z) = \langle yz, xz, xy + 2z\rangle$.

   (a) Given that $\mathbf{F}$ is conservative, find a potential function f for $\mathbf{F}$.

   (b) If C is the line segment from $(1, 0, -2)$ to $(4, 6, 3)$, use the fundamental theorem for line integrals to compute $\int_C \mathbf{F} \bullet d\mathbf{r}$.