## Line integrals of vector fields

1. Match each plot with one of the vector fields below.

Images created on https://kevinmehall.net/p/equationexplorer/vectorfield.html

(a) $\mathbf{F}(x, y)=\langle y, x\rangle$
(b) $\mathbf{F}(x, y)=\langle x, y\rangle$
(c) $\mathbf{F}(x, y)=\langle x,-y\rangle$
(d) $\mathbf{F}(x, y)=\langle x, 1\rangle$
2. Evaluate the line integral $\int_{C} \mathbf{F} \bullet d \mathbf{r}$ if $\mathbf{F}(x, y)=\left\langle x y, 3 y^{2}\right\rangle$ and $C$ is the curve given by $x=11 t^{4}, y=t^{3}$ with $0 \leqslant t \leqslant 1$.
3. Match each plot with one of the vector fields below. Image taken from Stewart (8th edition).

(a) $\mathbf{F}(x, y, z)=\langle 1.2,3\rangle$
(b) $\mathbf{F}(x, y, z)=\langle 1,2, z\rangle$
(c) $\mathbf{F}(x, y, z)=\langle x, y, 3\rangle$
(d) $\mathbf{F}(x, y, z)=\langle x, y, z\rangle$
4. Evaluate the line integral $\int_{C} \mathbf{F} \bullet d \mathbf{r}$ if $\mathbf{F}(x, y, z)=\langle x, y, x y\rangle$ and $C$ is the curve given by $x=\cos t$, $y=\sin t, z=t$ for $t \in[0, \pi)$.
5. Consider the force field $\mathbf{F}(x, y)=\left\langle x^{2}, x y\right\rangle$.
(a) If a particle moves once around the circle $x^{2}+y^{2}=4$ in the counter-clockwise direction, find the work done by the given force field on the particle.
(b) Does anything change if the particle moves around the circle in the clockwise direction instead?
6. Show that a constant force field does zero work on a particle that moves once around the circle $x^{2}+y^{2}=1$. Is the work still zero if the radius is not $1 ?$

