## Line integrals of vector fields

1. Match each plot with one of the vector fields below. Images created on https://kevinmehall.net/p/equationexplorer/vectorfield.html



- (a)  $\mathbf{F}(x,y) = \langle y,x \rangle$
- (b)  $\mathbf{F}(x,y) = \langle x,y \rangle$
- (c)  $\mathbf{F}(x,y) = \langle x, -y \rangle$
- (d)  $\mathbf{F}(x,y) = \langle x,1 \rangle$
- 2. Evaluate the line integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$  if  $\mathbf{F}(x, y) = \langle xy, 3y^2 \rangle$  and C is the curve given by  $x = 11t^4$ ,  $y = t^3$  with  $0 \leq t \leq 1$ .



3. Match each plot with one of the vector fields below. Image taken from Stewart (8th edition).

- (a)  $\mathbf{F}(x, y, z) = \langle 1.2, 3 \rangle$
- (b) **F** $(x, y, z) = \langle 1, 2, z \rangle$
- (c)  $\mathbf{F}(x, y, z) = \langle x, y, 3 \rangle$
- (d)  $\mathbf{F}(x, y, z) = \langle x, y, z \rangle$
- 4. Evaluate the line integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$  if  $\mathbf{F}(x, y, z) = \langle x, y, xy \rangle$  and C is the curve given by  $x = \cos t$ ,  $y = \sin t$ , z = t for  $t \in [0, \pi)$ .
- 5. Consider the force field  $\mathbf{F}(x, y) = \langle x^2, xy \rangle$ .
  - (a) If a particle moves once around the circle  $x^2 + y^2 = 4$  in the counter-clockwise direction, find the work done by the given force field on the particle.
  - (b) Does anything change if the particle moves around the circle in the clockwise direction instead?
- 6. Show that a constant force field does zero work on a particle that moves once around the circle  $x^2 + y^2 = 1$ . Is the work still zero if the radius is not 1?