Math 2110-003 Worksheet 15.8 **Name:** ____________________________ **Due:** 10/25/2017

Triple integrals in spherical coordinates

1. Using spherical coordinates, prove that the volume of a sphere of radius R is $\frac{4}{3}\pi R^3$.

2. Compute $\iiint\limits_E (x^2+y^2+z^2)^{1/2}\,dV$ where $E = \{(x,y,z) : x \leqslant 0, y \leqslant 0, z \geqslant 0, 4 \leqslant x^2+y^2+z^2 \leqslant 16\}$.

3. Compute $\iiint\limits_E x\,dV$ where $E = \{(x,y,z) : x \leqslant 0, z \leqslant 0, x^2+y^2+z^2 \leqslant 9\}$.

4. Sketch the solid whose volume is given by the iterated integral.

   (a) $\int_0^{2\pi}\int_0^{\pi/2}\int_0^3 \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$

   (b) $\int_0^{\pi/6}\int_0^{\pi/2}\int_0^3 \rho^2 \sin\phi\,d\rho\,d\theta\,d\phi$

   (c) $\int_0^{2\pi}\int_{\pi/2}^{\pi}\int_1^2 \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$

5. Compute the following integral by making a change in coordinates.

$$\int_{-2}^{2}\int_0^{\sqrt{4-y^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} x^2\sqrt{x^2+y^2+z^2}\,dz\,dx\,dy.$$