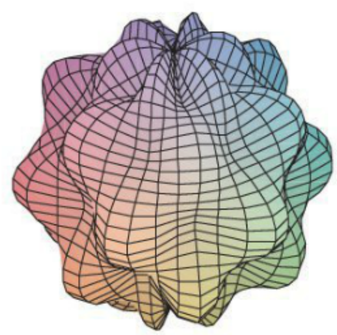
Math 2110-003 Worksheet 15.6 **Name:** __________________________ **Due:** 10/18/2017

Triple integrals (in Cartesian coordinates)

1. Compute the iterated triple integral.
   (a) $\displaystyle\int_{-1}^{2}\int_{0}^{1}\int_{0}^{3} xy + z^2\,dz\,dy\,dx$
   (b) $\displaystyle\int_{0}^{\pi/2}\int_{0}^{y}\int_{0}^{x} \cos(x+y+z)\,dz\,dx\,dy$
2. Compute $\displaystyle\iiint_E xy\,dV$ where $E = \{(x,y,z) : 0 \leqslant x \leqslant 3, 0 \leqslant y \leqslant x, 0 \leqslant z \leqslant x+y\}$.
3. Compute $\displaystyle\iiint_E x^2\,dV$ where E is the solid tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$.

Similar to what we have seen with single and double integrals, the triple integral $\iiint_E 1\,dV$ has a geometric meaning. Where $\int_I 1\,dx$ computed the length of the interval I and $\iint_D 1\,dA$ computed the area of the region D, now $\iiint_E 1\,dV$ computes the *volume* of the region E. This is a chief application of the triple integral as many volumes we can bound in space do not have a simple formula to compute their volume. On the right is a figure of what is referred to as a "bumpy sphere." This object has beeen used to model cancer tumors. To divine the volume of such a tumor, one must use multivariate calculus.



4. Find the volume of tetrahedron specified in problem 3.
5. Sketch the solid whose volume is given by the following iterated integral:
   (a) $\displaystyle\int_{0}^{1}\int_{0}^{1-x}\int_{0}^{2-2z} dy\,dz\,dx$
   (b) $\displaystyle\int_{0}^{2}\int_{0}^{2-y}\int_{0}^{4-y^2} \cos(x+y+z)\,dx\,dz\,dy$
6. Setup but do not evaluate an iterated triple integral which gives the volume bounded by $x^2+y^2+x^2 = 4$ and $z = \sqrt{x^2+y^2}$.
7. **Optional:** Find the volume specified in problem 6.

   Hint: View the region as type z and use polar coordinates to compute the resulting double integral. (Don't forget $r\,dr\,d\theta$!)
8. Write the other 5 integrals that are equivalent to

$$\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} f(x,y,z)\,dz\,dy\,dx.$$