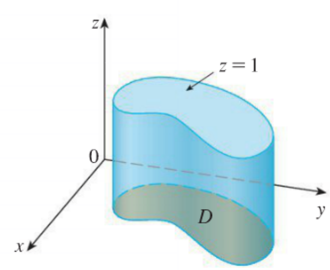
Math 2110-003 Worksheet 15.2 **Name:** ____________________ **Due:** 9/27/17

Double integrals over general regions

1. Evaluate $\int_0^1 \int_0^y x e^{y^3}\, dx\, dy$.
2. Find $\iint\limits_D x^2 y\, dA$ where D is the region bounded by $x = 0$, $y = 2$ and $x = y^2$.
3. Let D be the region in the xy-plane bounded by $x = 0$, $y = 1 - x$ and $y = 2 - 2x$.
   (a) Sketch the region D and say whether it is type-y, type-x, both or neither.
   (b) Set up, but do not evaluate, a double integral or sum of double integrals to integrate $f(x, y) = xy$ over the region D
4. Let D be the region in the first quadrant of the xy-plane given by $1 \leqslant x^2 + y^2 \leqslant 4$.
   (a) Sketch the region D and say whether it is type-y, type-x, both or neither.
   (b) Set up, but do not evaluate, a double integral or sum of double integrals to integrate $f(x, y) = xy$ over the region D
5. Sketch the region of integration and change the order of integration of $\int_1^2 \int_0^{\ln x} f(x, y)\, dy\, dx$.
6. Consider the double integral $\int_0^5 \int_y^5 e^{x^2}\, dx\, dy$.
   (a) Sketch the region being integrated over.
   (b) Rewrite the integral in the opposite order of integration, then evaluate. Notice that we cannot evaluate the integral in the original order.

Notice that $\int_a^b 1\, dx = b - a$, or the length of the interval $[a, b]$. Similarly, for double integrals $\iint_D 1\, dA = A(D)$ where $A(D)$ is the area of D in the xy-plane. This follows from the fact that $\iint_D 1\, dA$ is the volume contained beneath $z = 1$ and over the region D. (See the figure to the right: taken from Stewart's Calculus, 8th edition.) This volume is that of a cylinder of height 1 with base D. Thus it's volume is $1 \cdot A(D) = A(D)$. This idea will prove useful later when we compute volumes via triple integrals.



7. Compute $\iint\limits_R 1\, dA$ where $R = [0, 2] \times [-1, 4]$.
8. Compute $\iint\limits_D 3\, dA$ where D is the region bounded by the unit circle.
9. **Optional:** Use the properties of double integrals to verify that

   If $m \leqslant f(x, y) \leqslant M$ for all (x, y) in D, then
   $$mA(D) \leqslant \iint\limits_D f(x, y)\, dA \leqslant MA(D).$$
10. **Optional:** Use the previous problem to estimate $\iint\limits_D e^{\sin x \cos y}\, dA$ where D is the disk centered at the origin of radius 2.