Math 2110-003 Worksheet 15.1 **Name:** ____________________ **Due:** 9/27/17

Double integrals over rectangles

1. Evaluate each of the following double integrals:

   (a) $\displaystyle\iint_R (2x/y) - (1/\sqrt{y})\,dA$ where $R = [1,2] \times [1,4]$,

   (b) $\displaystyle\int_3^4 \int_1^5 \frac{x \ln y}{y}\,dx\,dy$

   (c) $\displaystyle\int_0^1 \int_0^{\pi/6} xy\cos(3x)\,dx\,dy$

2. (a) Approximate the volume of the solid that lies above the rectangle $R = [0,2] \times [0,2]$ and below the elliptic paraboloid $z = 16 - x^2 - 2y^2$ with a double Riemann sum by dividing R into four equal squares and using the upper right corner of each square as your sample points.

   Hint: It may help to draw a picture, both of R and the solid in question. Your Riemann sum should have four terms.

   (b) Find the exact volume of the solid mentioned above and find the error in your approximation.

3. (a) Find the volume contained beneath $z = 3x + 2y$ and above the xy-plane over the rectangle $0 \leqslant x \leqslant 1$, $0 \leqslant y \leqslant 2$.

   (b) Find the volume contained between the surfaces $z = 3x + 2y$ and $z = -x^2 - y^2$ over the same rectangle.

4. In single-variable calculus we learn that the *average value* of a function $f(x)$ on an interval $[a,b]$ is given by

$$f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

   This is simply the signed area under the graph of $f(x)$ on $[a,b]$ divided by the length of the interval. Notice in particular that the rectangle of length $b-a$ and height f_{ave} has the same area as the signed area under the graph of f:

$$(b-a)\cdot f_{\text{ave}} = \int_a^b f(x)\,dx.$$

   We can define a similar average value of a multi-variable function $f(x,y)$ over a rectangle R by setting

$$f_{\text{ave}} = \frac{1}{A(R)}\iint_R f(x,y)\,dA$$

   where $A(R)$ is the area of R.

   (a) Find the average value of $f(x,y) = x^2y^2$ over $R = [0,1] \times [0,2]$.

   (b) Give the dimensions of a rectangular prism with height f_{ave} whose volume is equal to the signed volume under the graph of x^2y^2 over R.

   (c) **Optional:** Can you find a square prism with height f_{ave} which has this same volume?