Math 2110-003 Worksheet 14.7 **Name:** ______________________________ **Due:** 9/27/17

Finding and classifying the extrema of a function

1. Find and classify all critical points of $f(x, y) = x^3 - 6xy - y^2$.
2. Find and classify all critical points of $f(x, y) = 5xye^{-y^2}$.
3. Find and classify all critical points of $f(x, y) = \sin(x)\sin(y)$ in the rectangle where $-\pi < x < \pi$ and $-\pi < y < \pi$.
4. Show that the function $f(x, y) = x^3$ has infinitely many critical points but the second derivative test fails at each one (i.e. $|H_f| = D = 0$ at each point).
5. True of false: if (a, b) is a critical point of $f(x, y)$ and both $f_{xx}(a, b) > 0$ and $f_{yy}(a, b) > 0$, then (a, b) must be a local minimum.
6. (a) Show that $(0, 0)$ is the only critical point of $f(x, y) = x^2 + 4xy + y^2$.
   (b) Show that both $f_{xx}(0, 0)$ and $f_{yy}(0, 0)$ are positive.
   (c) Classify the critical point $(0, 0)$.
   (d) How does this relate to question 5?
7. Find and classify all critical points of $f(x, y) = 3x - x^3 - 2y^2 + y^4$.
8. Find the shortest distance from the point $(1, 0, -2)$ to the plane $x + 2y + z = 4$.

   Hint: The distance between a point (x, y, z) and $(1, 0, -2)$ is given by $\sqrt{(x-1)^2 + y^2 + (z+2)^2}$. How can you ensure a point (x, y, z) and is on the plane and minimize this quantity simultaneously?