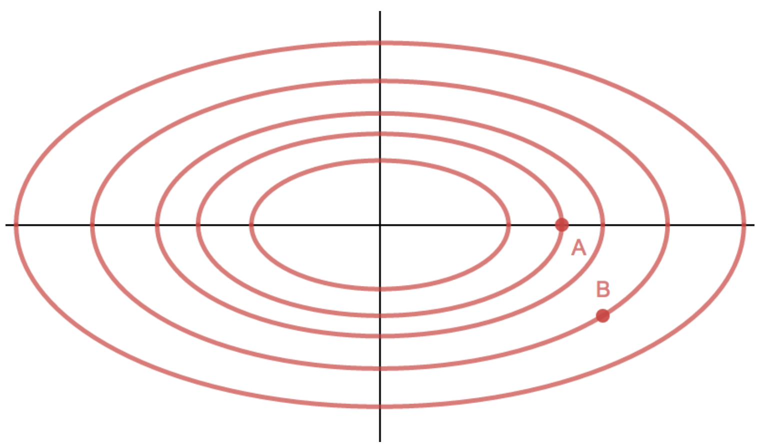
Math 2110-003 Worksheet 14.6 **Name:** ____________________________ **Due:** 9/27/17

The gradient vector and the directional derivative

1. Consider the function $f(x, y) = x^2y + 6xy$ at the point $(1, 2)$.
   (a) What is the gradient of $f(x, y)$?
   (b) Find directional derivative of f at $(1, 2)$ in the direction $\langle 3, 4\rangle$.
   (c) Find the direction of steepest ascent. Give your answer as a unit vector with this direction.
   (d) Find the maximum derivative of f at $(1, 2)$. That is, find the rate of change in the direction of steepest ascent.
   (e) What is the direction of steepest *descent*?
   (f) What is the minimum derivative of f at $(1, 2)$?
2. In general, for a function f the maximum derivative (a, b) is given by ___________ and is in the direction of ___________. Similarly, the minimum derivative at (a, b) s given by ___________ and is in the direction of ___________.
3. A contour plot of $f(x, y)$ is given below for $z = 0, 1, 2, 3, 4$ where the outermost level curve is $z = 0$ and the innermost is $z = 4$. Sketch the gradient vector at the two points A and B plotted on the level curves $z = 4$ and $z = 1$ respectively.



4. The function $A(x, y) = 4000 + 3xy - 4x^2 - 5y^2$ gives the altitude in feet at any point (x, y) on a hill (we can think of the (x, y) coordinates as specifying latitude and longitude). We are currently on the hill at $(-1, 2)$.
   (a) What is our current altitude?
   (b) If we begin moving in the direction of the vector $\langle 1, 7\rangle$, what will the initial slope be?
   (c) Find a vector (not necessarily unit) that points in a direction in which the initial slope will be 0.
5. Consider the function $h(r, s, t) = \ln(3r + 6s + 9t)$.
   (a) Find the directional derivative at $(1, 1, 1)$ in the direction of $\vec{\mathbf{v}} = \langle 4, 12, 6\rangle$.
   (b) What is the direction of the maximum directional derivative of h at $(1, 1, 1)$?
   (c) What is the maximum derivative?