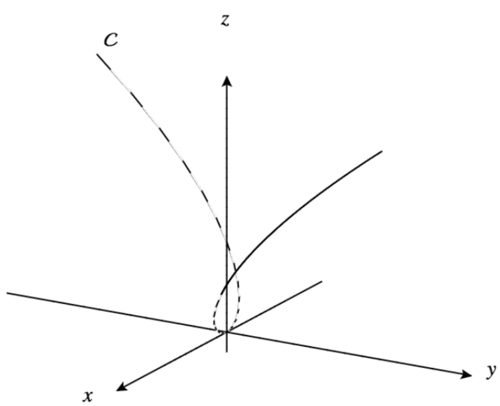
Vector valued functions part 2

1. A bee flies along the path $\mathbf{r}(t) = \langle 12t, 8t^{3/2}, 3t^2 \rangle$.
   (a) Find the displacement of the bee from $t = 0$ to $t = 1$.
   (b) Find the distance traveled by the bee from $t = 0$ to $t = 1$.

2. Consider the twisted cubic C given below. In class, we parametrized this particular curve by $\mathbf{r}_1(t) = \langle t, t^3, t^2 \rangle$ where $-2 \leqslant t \leqslant 2$.



Here we pay close attention to the solid portion of the curve.

This portion of the curve corresponds to the same vector function $\mathbf{r}_1(t) = \langle t, t^3, t^2 \rangle$ where t varies over a smaller domain, namely $1 \leqslant t \leqslant 2$.
There are many more ways to parameterize this curve though, for example

$$\mathbf{r}_2(t) = \langle e^u, e^{3u}, e^{2u} \rangle$$

where $0 \leqslant u \leqslant \ln 2$.

Does the choice of parametrization affect the arc length of the curve?

   (a) Setup, but do not evaluate the arc length of C with respect to both parametrization $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$. (You should get two integrals.)
   (b) To show that parametrization does not affect arc length, show that the two integrals you found in the previous part are equal. This can be done by performing an appropriate change of variables.

In Calculus II, we saw that the arc length of the graph of a function $f(x)$ from $x = a$ to $x = b$ was given by $\int_a^b \sqrt{1 + (f'(x))^2}\, dx$. This in fact, follows from what we have learned about arc length here.

3. (a) Parametrize the graph of a function $y = f(x)$ from $x = a$ to $x = b$ as a space curve in $\mathbb{R}^3$
   (b) Find the arc length of the curve you parametrized in part (i).

4. Consider the vector function $\mathbf{r}(t) = \langle 5 - 2t, 4t - 3, 4t \rangle$.
   (a) Find the arc length function for the curve measured from the point $P = (5, -3, 0)$ in the direction of increasing t.
   (b) Reparametrize the curve with respect to arc length starting from P.
   (c) Find the point 3 units along the curve (in the direction of increasing t) from P.

5. We don't know the equation that defines a specific surface S, but we are able to determine the equation of two curves that lie on the surface that intersect at the point $(2, 1, 3)$, namely

$$\mathbf{r}_1(t) = \langle 2 + 3t, 1 - t^2, 3 - 4t + t^2 \rangle \qquad \text{and} \qquad \mathbf{r}_2(t) = \langle 1 + t^2, 2t^3 - 1, 2t + 1 \rangle.$$

   Determine an equation for the tangent plane at the point $(2, 1, 3)$.

   Note: This allows us to approximate values on the surface we have no equation for.