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## Vector-valued functions part 1

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1. Give a vector function for each of the following curves, including the appropriate range of  $t$ -values.
  - (a) A circle in the plane  $x = 4$  centered at  $(4, 0, 0)$  with radius 3, traced once.
  - (b) The portion of the curve in the  $xy$ -plane with equation  $y = \sqrt{x+1}$  from  $(0, 1, 0)$  to  $(3, 2, 0)$ .
  - (c) The portion of the curve in the plane  $y = -1$  with equation  $x = z^2 - z + 2$  from  $(2, -1, 1)$  to  $(8, -1, 3)$ .
2. Consider the curve given by the vector function  $\mathbf{r}(t) = \langle t^2, 1 - 3t, 1 + t^3 \rangle$ .
  - (a) Find the value(s) of  $t$  for which the curve passes through the points  $(1, 4, 0)$  and  $(9, -8, 28)$ .
  - (b) Show that the curve does not pass through the point  $(4, 7, -6)$ .

We say that two curves  $\mathbf{r}_1(t)$  and  $\mathbf{r}_2(t)$  *intersect* if they ever pass through the same point and that they *collide* if they cross at the same time.

For example, the curves  $\mathbf{r}_1(t) = \langle t - 1, 0 \rangle$  and  $\mathbf{r}_2(t) = \langle \cos t, \sin t \rangle$  intersect at points  $(-1, 0)$  and  $(1, 0)$  but they do not collide. Before continuing, reflect upon why this is the case.

3. Two curves  $\mathbf{r}_1(t)$  and  $\mathbf{r}_2(s)$  intersect if there are  $t_0$  and  $s_0$  such that  $\mathbf{r}_1(t_0) = \mathbf{r}_2(s_0)$ . These curves collide here if \_\_\_\_\_.
4. Suppose two missiles are fired with trajectories given by the vector functions

$$\mathbf{r}_1(t) = \langle (t-4)^2, t^2 - 8t + 34, (t-4)^2 \rangle \quad \text{and} \quad \mathbf{r}_2(t) = \langle 4t - 19, (t-4)^3, 5t - 26 \rangle$$

Assuming  $t \geq 0$ , will these missiles collide? If so, when?

*Hint:* It may be helpful to rename the variable in the second trajectory as  $s$ , namely  $\mathbf{r}_2(s) = \langle 4s - 19, (s-4)^3, 5s - 26 \rangle$ .

5. Find an equation for the line tangent to the curve  $\mathbf{r}(t) = \langle e^t, te^t, te^{t^2} \rangle$  at the point  $(1, 0, 0)$ .