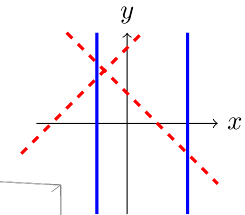
Name: ____________________________

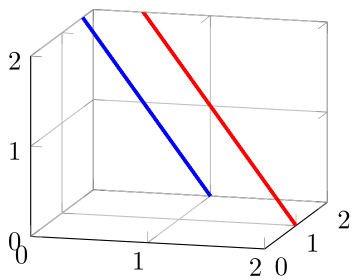
§ 12.5: Intersections of lines and planes

Due: Beginning of class, 9/13

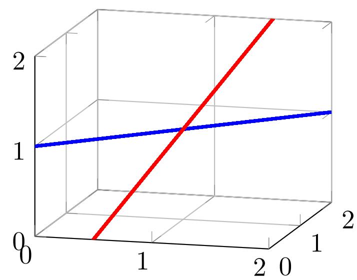
In two dimensions, lines are always either *parallel* (solid) or *intersecting* (dashed). See the figure to the right.



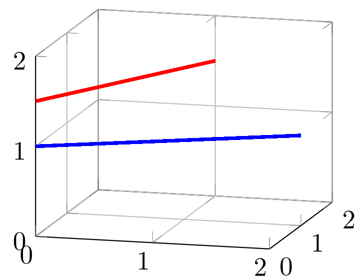
In three dimensions however this is not always the case. Lines can be parallel (as seen below and to the left), intersecting (as seen below center) or *neither* , in which case we call them *skew lines* (see below right).



$\vec{\mathbf{r}}_1(t) = \langle -t+1, t+2, t\rangle$
$\vec{\mathbf{r}}_2(t) = \langle -t+2, t+1, t\rangle$



$\vec{\mathbf{r}}_1(t) = \langle t+1, t+1, 1\rangle$
$\vec{\mathbf{r}}_2(t) = \langle t+1, 2t+2, 2t+1\rangle$



$\vec{\mathbf{r}}_1(t) = \langle 2t, t, 1\rangle$
$\vec{\mathbf{r}}_2(t) = \langle t, 2t, 1.5\rangle$

Here we wish to determine in which case a given pair of lines fall.

First, to determine if lines are parallel, we consider their direction vectors. What can we say about the direction vectors of two parallel lines?

Are $x(t) = 1+3t, y(t) = 2+t, z(t) = 17$ and $\vec{\mathbf{r}}(s) = \langle -3s, 1-3s, 11\rangle$ parallel lines? How do you know?

We now consider lines that intersect at a point. If two lines interesect at one point, can they be parallel? What does this tell us about their direction vectors?

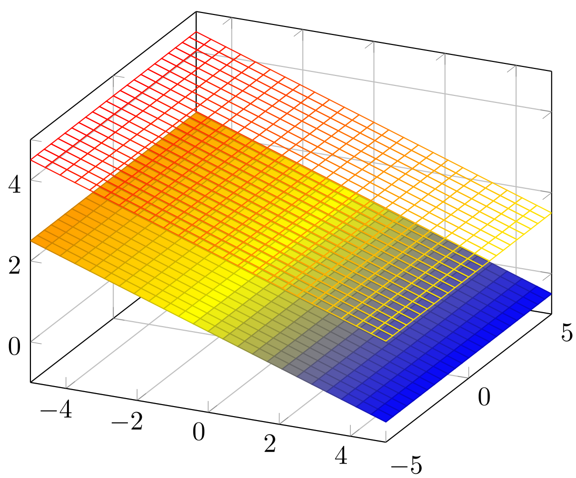
If two lines, say $\vec{\mathbf{r}}_1(t)$ and $\vec{\mathbf{r}}_2(s)$, intersect at a point, then there must be a pair of numbers t_0 and s_0 such that $\vec{\mathbf{r}}_1(t_0) = \vec{\mathbf{r}}_2(s_0)$.

The lines $\vec{\mathbf{r}}_1(t) = \langle 1+t, 2+t, 1-t\rangle$ and $\vec{\mathbf{r}}_2(s) = \langle s, 2s+1, 2+s\rangle$ intersect. Find the point at which this occurs.

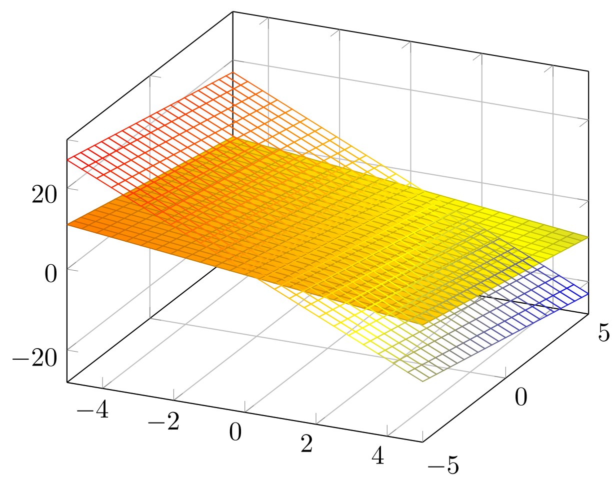
Finally, if lines are not parallel and are not intersecting what must they be?

Determine if $x(t) = t, y(t) = 2 - t, z(t) = 5 + t$ and $x(s) = 2s, y(s) = 3 + s, z(s) = 1 - 3s$ are parallel, intersecting or skew lines.
(*Hint:* It may help to answer the questions "Are they parallel?" "Do they intersect?" "Are they skew?" in that order.)

We now turn are attention to the intersections of planes. Planes can be parallel (left) or intersecting (right). Planes in 3D, much line likes in 2D, cannot be skew.



$.3x + z = 1$
$.3x + z = 3$



$x + y + z = 1$
$4x + y + z = 2$

If two planes are parallel, how do their normals relate?

Determine if the planes $6(x-1) + 3(y-6) + 9(z-2) = 17$ and $-2x - y - 3z = -3$ intersect or are parallel.

Finally, we consider the utility of finding the x-, y- and z-intercepts of a plane to help us sketch it. Here the x-intercept is where the plane crosses the x-axis

Find the x-, y- and z-intercepts of $4x + y + 2z = 4$ and sketch the portion of this plane in the first octant.