§ 12.3 and 12.4 The dot and cross products:

Due: Beginning of class, 9/6

1. How do $\hat{\mathbf{a}} \bullet \hat{\mathbf{a}}$ and $|\hat{\mathbf{a}}|$ relate?

2. Find the angle between $\langle \sqrt{3}, 1 \rangle$ and $\langle 1, \sqrt{3} \rangle$.

3. Give a formula for the angle between two vectors $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$.

4. Suppose two vectors are orthogonal, what does this tell you about their dot product?

5. If $\hat{\mathbf{a}} \bullet \hat{\mathbf{b}} = 0$, what can you say about the angle between $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$?

   Fact: Two vectors are orthogonal if and only if their dot prouduct is ______.

6. Suppose $\hat{\mathbf{a}} \bullet \hat{\mathbf{b}} = \pm|\hat{\mathbf{a}}||\hat{\mathbf{b}}|$. What can you say about $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$? What is the angle between $\langle 2, 4 \rangle$ and $\langle -1, -2 \rangle$?

7. Find the area of the parallelogram with points $(0,0,0),(1,2,1),(2,2,2)$ and $(3,4,3)$.

8. Suppose $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are parallel, what can you say about the area of the parallelogram they span? What about $|\hat{\mathbf{a}} \times \hat{\mathbf{b}}|$?

Fact: Two vectors $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are parallel if and only if $\hat{\mathbf{a}} \times \hat{\mathbf{b}} =$ ______.

9. Verify 4 of the following facts:

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \qquad \hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}} \qquad \hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}$$

$$\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}} \qquad \hat{\mathbf{k}} \times \hat{\mathbf{j}} = -\hat{\mathbf{i}} \qquad \hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$$

10. For vectors $\hat{\mathbf{a}} = \langle a_1, a_2, a_3 \rangle$ and $\hat{\mathbf{b}} = \langle b_1, b_2, b_3 \rangle$, prove that $\hat{\mathbf{a}} \times \hat{\mathbf{b}}$ is indeed orthogonal to $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ by computing

$$(\hat{\mathbf{a}} \times \hat{\mathbf{b}}) \bullet \hat{\mathbf{a}} \text{ and } (\hat{\mathbf{a}} \times \hat{\mathbf{b}}) \bullet \hat{\mathbf{b}}.$$

(It may help to try this with a concrete case or two.)