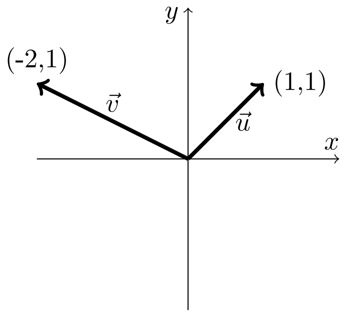
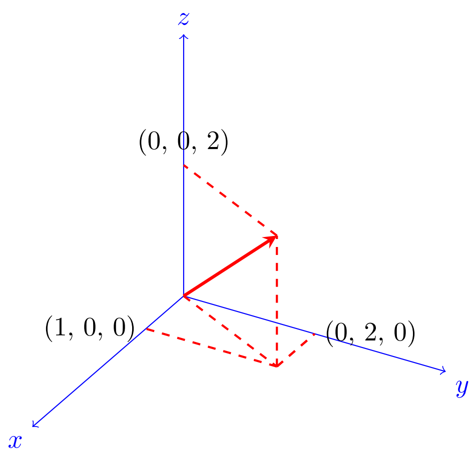
§ 12.2 Vectors: Lengths, unit vectors, and a few properties

Due: Beginning of class, 9/6

Given a vector's components, $\hat{\mathbf{a}} = \langle a_1, a_2, a_3 \rangle$, it is easy to determine its direction in $\mathbb{R}^3$ but how might we determine the other defining property of a vector, its *magnitude*?



Determine the lengths of $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$. We denote these $|\hat{\mathbf{u}}|$ and $|\hat{\mathbf{v}}|$.



Determine the length of the vector $\hat{\mathbf{w}}$ depicted on the left.

Give formulas to compute the length of any two dimensional or three dimensional vector. For example, if $\hat{\mathbf{a}} = \langle a_1, a_2 \rangle$ and $\hat{\mathbf{b}} = \langle b_1, b_2, b_3 \rangle$, find

$$|\hat{\mathbf{a}}| =$$

$$|\hat{\mathbf{b}}| =$$

We call vectors with length 1 **unit vectors**. For example, the three vectors

$$\hat{\mathbf{i}} = \langle 1, 0, 0 \rangle \quad \hat{\mathbf{j}} = \langle 0, 1, 0 \rangle \quad \hat{\mathbf{k}} = \langle 0, 0, 1 \rangle$$

are all unit vectors. These play a special role and are called the **standard basis vectors.** Sketch these to the right.

The role these basis vectors play are to decompose any vector into a sum of these three. For example,

$$\begin{aligned}\langle 2,\pi,7\rangle &= \langle 2,0,0\rangle + \langle 0,\pi,0\rangle + \langle 0,0,7\rangle \\ &= 2\langle 1,0,0\rangle + \pi\langle 0,1,0\rangle + 7\langle 0,0,1\rangle = 2\hat{\mathbf{i}} + \pi\hat{\mathbf{j}} + 7\hat{\mathbf{k}}\end{aligned}$$

Suppose $\hat{\mathbf{c}} = 2\hat{\mathbf{i}} + 7\hat{\mathbf{k}}$ and $\hat{\mathbf{d}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$. Express $3\hat{\mathbf{c}} + 2\hat{\mathbf{d}}$ in terms of $\hat{\mathbf{i}}, \hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$.

There are, of course, many more unit vectors than simply $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$. For example $\langle 1/\sqrt{2}, 1/\sqrt{2}, 0\rangle$.
In fact, for any vector $\hat{\mathbf{a}}$, we can find a unit vector $\hat{\mathbf{u}}$ pointing in the same direction.

As $(1/|\hat{\mathbf{a}}|)$ is a postive scalar, we have that $\hat{\mathbf{u}} = \dfrac{\hat{\mathbf{a}}}{|\hat{\mathbf{a}}|}$ points in the same direction as $\hat{\mathbf{a}}$. Show that $\hat{\mathbf{u}}$ is a unit vector.

Find a unit vector pointing in the opposite direction of $2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$.

Below are several properties of vectors. Verify any 3 of these.

If $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$ and $\hat{\mathbf{c}}$ are vectors in $\mathbb{R}^2$ or $\mathbb{R}^3$, and c and d are scalars then

1. $\hat{\mathbf{a}} + \hat{\mathbf{b}} = \hat{\mathbf{b}} + \hat{\mathbf{a}}$	2. $\hat{\mathbf{a}} + \hat{\mathbf{b}} + \hat{\mathbf{c}} = \hat{\mathbf{a}} + \hat{\mathbf{b}} + \hat{\mathbf{c}}$	3. $\hat{\mathbf{a}} + \hat{\mathbf{0}} = \hat{\mathbf{a}}$
4. $\hat{\mathbf{a}} + (-\hat{\mathbf{a}}) = 0$	5. $c(\hat{\mathbf{a}} + \hat{\mathbf{b}}) = c\hat{\mathbf{a}} + c\hat{\mathbf{b}}$	6. $(c+d)\hat{\mathbf{a}} = c\hat{\mathbf{a}} + d\hat{\mathbf{a}}$
7. $(cd)\hat{\mathbf{a}} = c(d\hat{\mathbf{a}})$	8. $1\hat{\mathbf{a}} = \hat{\mathbf{a}}$	9. $|c\hat{\mathbf{a}}| = |c||\hat{\mathbf{a}}|$