Math 2110Q: Helpful Formulas

1. **The Second Derivative Test**

   Let (a, b) be a critical point of a function $f(x, y)$ with $D(x, y) = f_{xx}f_{yy} - f_{xy}^2$.

   1. If $D(a, b) > 0$, then (a, b) is either a local maximum or minimum
      (a) $f_{xx}(a, b) < 0 \Rightarrow (a, b)$ is a local maximum
      (b) $f_{xx}(a, b) > 0 \Rightarrow (a, b)$ is a local minimum
   2. $D(a, b) < 0 \Rightarrow (a, b)$ is a saddle
   3. $D(a, b) = 0 \Rightarrow$ the test is inconclusive

2. **Summary of Line Integrals and Surface Integrals**

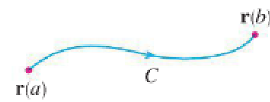
LINE INTEGRALS	SURFACE INTEGRALS
$C : \vec{r}(t),\ a \leqslant t \leqslant b$	$S : \vec{r}(u, v),\ (u, v) \in D$
$ds = \lvert\vec{r}\,'(t)\rvert\ dt$ =arc length differential	$dS = \lvert\vec{r}_u \times \vec{r}_v\rvert\ dA$= surface area differential
$\int_C f(x, y, z)\ ds = \int_a^b f(\vec{r}(t))\lvert\vec{r}\,'(t)\rvert\ dt$ (independent of orientation of C)	$\iint_S f(x, y, z)\ dS = \iint_D f(\vec{r}(u, v))\lvert\vec{r}_u \times \vec{r}_v\rvert\ dA$ (independent of orientation of S)
$d\vec{r} = \vec{r}\,'(t)\ dt$	$d\vec{S} = (\vec{r}_u \times \vec{r}_v)\ dA$
$\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\ dt$ (depends on orientation of C)	$\iint_S \vec{F} \cdot d\vec{S} = \iint_D \vec{F}(\vec{r}(u, v)) \cdot (\vec{r}_u \times \vec{r}_v)\ dA$ (depends on orientation of S)
Theorems that *may* apply: Fundamental Theorem for Line Integrals Green's Theorem	Theorems that *may* apply: Stokes' Theorem Divergence Theorem

Fundamental Theorem of Calculus

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

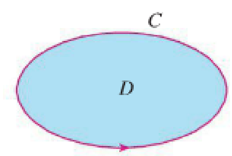
Fundamental Theorem for Line Integrals

$$\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$$



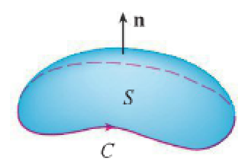
Green's Theorem

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \int_C P\,dx + Q\,dy$$



Stokes' Theorem

$$\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{r}$$



Divergence Theorem

$$\iiint_E \operatorname{div} \mathbf{F}\,dV = \iint_S \mathbf{F} \cdot d\mathbf{S}$$