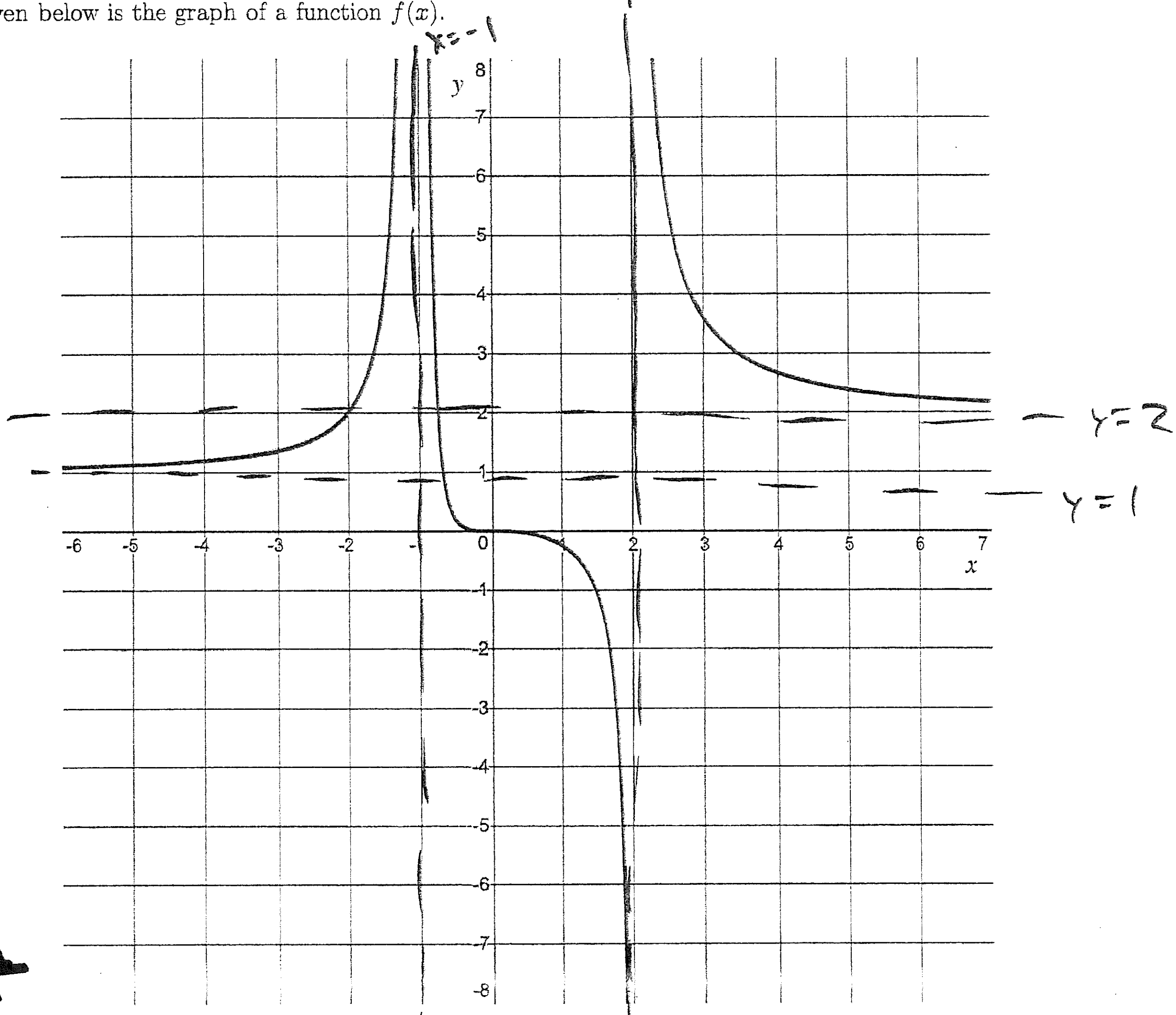
Calc β, BRIDGE 2018 Name: Key Due: Tuesday, July 10

Homework #4: Limits and infinity

Note: Your work can only be assessed if it is legible. [redacted] You do not need a calculator to complete this assignment.

1. Given below is the graph of a function $f(x)$.



[redacted]

(a) Specify the vertical asymptotes of $y = f(x)$ and justify your statements with an appropriate statement regarding limits.

[redacted] $x=1$ b/c $\lim_{x\to 1} f(x) = -\infty$ or $\lim_{x\to 1} f(x) = \infty$. $x=-1$ b/c $\lim_{x\to -1} f(x) = \infty$

(b) Specify the horizontal asymptotes of $y = f(x)$ and justify your statements with an appropriate statement regarding limits.

$\lim_{x\to -\infty} f(x) = 1$ so $y=1$ and $y=2$ b/c $\lim_{x\to\infty} f(x) = 2$.

(c) *Bonus:* Give an expression for a possible function $f(x)$ which might have this graph.

[redacted]

$$f(x) = \begin{cases} \dfrac{x^3}{(x+1)^2(x-2)} & x<2 \\ \dfrac{2x}{x-2} & x>2 \end{cases}$$

[redacted]

2. Compute the following limits or explain why they do not exist (and if it approaches ∞ or $-\infty$.)

(a) $\lim_{x\to 1^+} \frac{x-2}{x-1} = -\infty$

Top is negative, near 1.

Bottom is positive, near 0.

Whole ratio is negative, very large so

(b) $\lim_{x\to\infty} \frac{2x+3}{6x-7} = \lim_{x\to\infty} \frac{2+\frac{3}{x}}{6-7/x} = \frac{2}{6} = \frac{1}{3}.$

(c) $\lim_{x\to 0} \frac{1}{x} - \frac{1}{x^2} = \lim_{x\to 0} \frac{x-1}{x^2} = -\infty.$

top is close to -1,

bottom is pos and

small so

(d) $\lim_{x\to\infty} \frac{x^3}{\sqrt{6x^4-1}} = \lim_{x\to\infty} \frac{\frac{x^3}{x^2}}{\sqrt{\frac{6x^4-1}{x^4}}} = \lim_{x\to\infty} \frac{x}{\sqrt{6-\frac{1}{x^4}}} = \infty$

top is pos. larger and larger,

bottom is close to $\sqrt{6}$

so

(e) $\lim_{x\to-\infty} \frac{4x^3+6x^2-2}{2x^3-4x+5}$ $= \lim_{x\to-\infty} \frac{4+\frac{6}{x}-\frac{2}{x^3}}{2-\frac{4}{x^2}+\frac{5}{x^3}} = \frac{4+0-0}{2-0+0} = 2$

(f) $\lim_{x\to\infty} \sqrt{x^2+1}-x$.
Hint: Multiply the expression by 1 in the form of the conjugate radical.

$$\lim_{x\to\infty} \sqrt{x^2+1}-x\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}\right) = \lim_{x\to\infty} \frac{x^2+1-x^2}{\sqrt{x^2+1}+x}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{x^2+1}+x} = 0.$$

top close to 1, bottom larger and larger

[redacted]

3. (a) T/F (with justification) The line $x = 1$ is a vertical asymptote of the graph $y = \frac{x^2-1}{x^2-2x+1}$.

$$\lim_{x\to1^+} \frac{x^2-1}{x^2-2x+1} = \lim_{x\to1^+} \frac{(x+1)(x-1)}{(x-1)^2} = \lim_{x\to1^+} \frac{x+1}{(x-1)} = \infty$$

b/c top goes to 2, while bottom is pos. near 0.

So $\boxed{T}$

(b) T/F (with justification) The line $x = 1$ is a vertical asymptote of the graph $y = \frac{x^2-2x+1}{x^2-1}$.

Here $\lim_{x\to1^\pm} \frac{x^2-2x+1}{x^2-1} = \lim_{x\to1^\pm} \frac{(x-1)^2}{(x+1)(x-1)} = \lim_{x\to1^\pm} \frac{x-1}{x+1} = 0.$

So $\boxed{F}$ as $\lim_{x\to1^\pm} f(x) \neq \pm\infty$.

4. Consider the function $f(x) = \dfrac{x}{\sqrt{4+2x^2}}$.

[redacted] (a) Compute $\lim_{x\to\infty} f(x)$.

$x > 0$ so $x = \sqrt{x^2}$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x}{\sqrt{4+2x^2}} = \lim_{x\to\infty} \frac{1}{\frac{\sqrt{4+2x^2}}{x}} = \lim_{x\to\infty} \frac{1}{\sqrt{\frac{4}{x^2}+2}}$$

$$= \frac{1}{\sqrt{2}}.$$

[redacted] (b) Compute $\lim_{x\to-\infty} f(x)$.

$x < 0$ so $-x = \sqrt{x^2}$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{\frac{x}{-x}}{\frac{\sqrt{4+2x^2}}{\sqrt{x^2}}} = \lim_{x\to-\infty} \frac{-1}{\sqrt{\frac{4}{x^2}+2}} = \frac{-1}{\sqrt{2}}.$$

[redacted] (c) What are the horizontal asymptotes of $y = f(x)$?

$y = \frac{1}{\sqrt{2}}$

and

$y = \frac{-1}{\sqrt{2}}$

[redacted] (d) Does $f(x)$ have any vertical asymptotes? Justify your answer.

No, $f(x)$ does not tend towards $\pm\infty$ as x does so we would need asymptotic behavior at some input $x=a$. Since the denominator of $f(x)$ is always positive this will never occur.

[redacted] (e) Based upon your previous work, sketch a possible graph of $f(x)$.