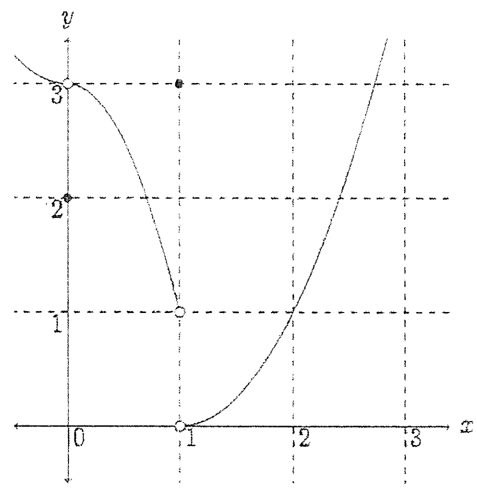
Calc β, BRIDGE 2018 Name: Key Due: [redacted] Thursday July 5

Homework #2: Limits and continuity

Note: Your work can only be assessed if it is legible. You must show all of you work on all problems save 1,5
You do not need a calculator to complete this assignment.

[redacted]

1. The graph of $y = f(x)$ is below. Use it to compute each limit or explain why it doesn't exist.



(a) $\lim_{x\to 0^-} f(x) = 3$

(b) $\lim_{x\to 1^-} f(x) = 1$

(c) $\lim_{x\to 2^-} f(x) = 1$

(d) $\lim_{x\to 0^+} f(x) = 3$

(e) $\lim_{x\to 1^-} f(x) = 0$

(f) $\lim_{x\to 2^+} f(x) = 1$

(g) $\lim_{x\to 0} f(x) = 3$

(h) $\lim_{x\to 1} f(x)$ DNE. Left and right limits aren't equal.

(i) $\lim_{x\to 2} f(x) = 1$

(j) $f(0) = 2$

(k) $f(1) = 3$

(l) $f(2) = 1$

[redacted]

2. T/F (with justification) If $\lim_{x\to 2} g(x) = 0$ and $\lim_{x\to 2} h(x) = 0$ then $\lim_{x\to 2} \frac{g(x)}{h(x)}$ does not exist.

$\boxed{F}$ If $g(x) = h(x) = x-2$,

1 pt.

$$\lim_{x\to 2} \frac{g(x)}{h(x)} = \lim_{x\to 2} \frac{x-2}{x-2} = \lim_{x\to 2} 1 = 1.$$

Thus, the limit exists.

2 pts

[redacted]

3. Evaluate the following limits exactly using algebra and limit laws.

(a) $\lim_{x \to 9} \dfrac{x-9}{\sqrt{x}-3}$

$$\lim_{x \to 9} \frac{x-9}{\sqrt{x}-3} \cdot \left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right) = \lim_{x \to 9} \frac{(x-9)(\sqrt{x}+3)}{(x-9)} = \lim_{x \to 9} \sqrt{x}+3$$

$$= \boxed{6}$$

(b) $\lim_{x \to -4} \dfrac{x^2+4x}{x^2+3x-4}$

$$\lim_{x \to -4} \frac{x^2+4x}{x^2+3x-4} = \lim_{x \to -4} \frac{x(x+4)}{(x+4)(x-1)} = \lim_{x \to -4} \frac{x}{x-1} = \frac{-4}{-5} = \boxed{\frac{4}{5}}$$

4. The squeeze theorem will prove valuable in this problem.

[redacted] (a) Evaluate $\lim_{x \to 1} (x-1)^4 \cos\left(\dfrac{1}{1-x}\right)$.

$-1 \le \cos\left(\frac{1}{1-x}\right) \le 1$ so $-(x-1)^4 \le (x-1)^4 \cos\left(\frac{1}{1-x}\right) \le (x-1)^4$

and $\lim_{x \to 1} -(x-1)^4 = \lim_{x \to 1} (x-1)^4 = 0$ so by the squeeze theorem,

$$\lim_{x \to 1} (x-1)^4 \cos\left(\frac{1}{1-x}\right) = 0$$

[redacted] (b) Bonus: Let

$$f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational.} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

Show that $\lim_{x \to 0} f(x) = 0$.

Notice for any x, $0 \le f(x) \le x^2$.

Since $\lim_{x \to 0} 0 = \lim_{x \to 0} x^2 = 0$, the squeeze theorem guarentees that $\lim_{x \to 0} f(x) = 0$.

[redacted]

5. Let [redacted]

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1, \\ 4 & \text{if } x = 1, \\ x + 2 & \text{if } 1 < x \leqslant 2, \\ 6 - x & \text{if } x > 2. \end{cases}$$

Evaluate the following limits if they exist. If a limit does not exist, write DNE.

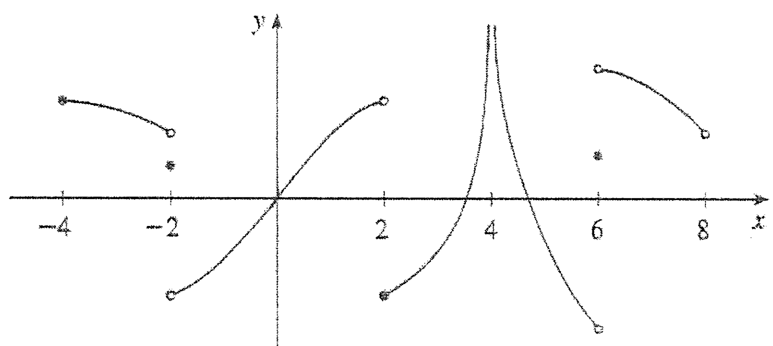
(a) $\lim_{x \to 1^-} f(x) = 2$ (g) $\lim_{x \to 2^-} f(x) = 4$

(b) $\lim_{x \to 1^+} f(x) = 3$ (h) $\lim_{x \to 2^+} f(x) = 4$

(c) $\lim_{x \to 1} f(x)$ DNE (i) $\lim_{x \to 2} f(x) = 4.$

6. The graph of a function $g(x)$ is given below. State the intervals on which g is continuous.

[redacted]



$(-4, 2), (-2, 2), (2, 4), (4, 6), (6, 8)$

[redacted]

7. Is

$$f(x) = \begin{cases} \sin x & \text{if } x \leqslant 0, \\ 1 + \cos x & \text{if } x > 0. \end{cases}$$

continuous on the interval $(-1, 1)$?

No. $f(0) = \sin(0) = 0$ but $\lim_{x \to 0} f(x)$ DNE

because $\lim_{x \to 0^-} f(x) = \sin 0 = 0$

and $\lim_{x \to 0^+} f(x) = 1 + \cos 0 = 2.$

The one-sided limits aren't equal.

[redacted] 8. For what value of the constant c is the function f continuous on the entire real line $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2, \\ x^3 - cx & \text{if } x \geqslant 2. \end{cases}$$

Here, $\lim\limits_{x \to 2^-} f(x) = c(2)^2 + 2 \cdot 2 = 4(c+1) = 4c+4$

$\lim\limits_{x \to 2^+} f(x) = 2^3 - c(2) = 8 - 2c.$

So $\lim\limits_{x \to 2} f(x)$ exists if c is s.t. $4c+4 = 8-2c$

$\Longrightarrow \; 6c = 4 \; \Rightarrow \; c = \frac{2}{3}.$

Then $\lim\limits_{x \to 2} f(x) = 4\left(\frac{2}{3}\right) + 4$
$= 8 - 2\left(\frac{2}{3}\right) = \frac{8}{3} + 4 = \frac{20}{3}.$

And $f(2) = \left(\frac{2}{3}\right)2^2 + 2(2) = \left(\frac{2}{3}\right) \cdot 4 + 4 = \frac{8}{3} + 4 = \frac{20}{3}.$

So $\lim\limits_{x \to 2} f(x) = f(2)$ and thus f is continuous.

[redacted] 9. Use the intermediate value theorem to show that there is a solution to $x - \sqrt{x} - \ln x = 0$ on the interval $[2, 3]$. Explain your reasoning.

Hint: You may find the following facts helpful. $\sqrt{2} \geqslant 1.41$ $\quad \ln 2 \geqslant 0.69$
$\sqrt{3} \geqslant 1.74$ $\quad \ln 3 \geqslant 1.1$

x, $\sqrt{x}$ and $\ln x$ are all continuous on $(0, \infty)$ so

$x - \sqrt{x} - \ln x$ is continuous on $(2, 3)$.

Notice $2 - \sqrt{2} - \ln 2 = 2 - (\sqrt{2} + \ln 2) \leq 2 - (1.41 + 0.69)$
$= 2 - 2.1 = -0.1 < 0$

and $3 - \sqrt{3} - \ln 3 = 3 - (\sqrt{3} + \ln 3) \geq 3 - (1.74 + 1.1)$
$= 3 - 2.84 > 0$

So, by the IVT there is a $2 \leq c \leq 3$ s.t. $c - \sqrt{c} - \ln c = 0$.