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Calc β, BRIDGE 2018 Name: Key Due: Tuesday, July 3

Homework #1: Warming up for Calculus

Note: Your work can only be assessed if it is legible. You do not need a calculator to complete this assignment.

1. Simplify the following algebraic expressions. Be sure to show the algebraic steps required to find the final answer.

(a) Simplify $\frac{1}{3^{-2}} - \frac{1}{3} + \frac{1}{4^{-1}}$.

$\frac{1}{3^{-2}} - \frac{1}{3} + \frac{1}{4^{-1}} = 9 + 4 - \frac{1}{3} = 13 - \frac{1}{3} = \boxed{\frac{38}{3}}$

(b) Simplify $\frac{(x^2y^{-3})^2}{(y^{-3}x^{-2})^{-2}}$.

$\frac{(x^2y^{-3})^2}{(y^{-3}x^{-2})^{-2}} = \frac{x^4 y^{-6}}{x^4 y^6} = \boxed{\frac{1}{y^{12}}}$

(c) Simplify $(4x^6)^{3/2}$.

$(4x^6)^{3/2} = (2x^3)^3 = \boxed{8x^9}$

(d) Let $f(x) = x^2 + 2x$ and $h \neq 0$. Simplify $\frac{f(x+h) - f(x)}{h}$, the *difference quotient* of $f(x)$.

$f(x+h) = (x+h)^2 + 2(x+h) = x^2 + 2xh + h^2 + 2x + 2h$,

$f(x) = x^2 + 2x \implies f(x+h) - f(x) = 2xh + h^2 + 2h$

$\implies \frac{f(x+h) - f(x)}{h} = \boxed{2x + h + 2}$

(e) Rationalize $\frac{3}{x - \sqrt{x}}$.

$\frac{3}{x - \sqrt{x}} \cdot \left(\frac{x + \sqrt{x}}{x + \sqrt{x}}\right) = \boxed{\frac{3(x + \sqrt{x})}{x^2 - x}}$

2. Factor the following expressions fully.

(a) $x^2 - 9$

(b) $x^2 - 2x - 35$

(c) $x^3 - a^2x$

a) $x^2 - 9 = (x+3)(x-3)$

b) $x^2 - 2x - 35 = (x-7)(x+5)$

c) $x^3 - a^2x = x(x-a)(x+a)$

3. Simplify the following exponential functions.

(a) $\frac{2^{5x}}{2^x}$

$$\frac{2^{5x}}{2^x} = 2^{5x} \cdot 2^{-x} = 2^{5x-x} = \boxed{2^{4x}}$$

(b) $e^{2x}e^{-3x}$

$$e^{2x} \cdot e^{-3x} = e^{2x-3x} = e^{-x} = \boxed{\frac{1}{e^x}}$$

(c) $\frac{e^{2x}-1}{e^x-1}$

$$\frac{e^{2x}-1}{e^x-1} = \frac{(e^x)^2-(1)^2}{e^x-1} = \frac{(e^x-1)(e^x+1)}{(e^x-1)} = \boxed{e^x+1}$$

(d) $\sqrt[3]{5^{9x}}$

$$\sqrt[3]{5^{9x}} = (5^{9x})^{1/3} = \boxed{5^{3x}}$$

4. Evaluate $\log_4\left(\frac{1}{64}\right)$.

$$\log_4\left(\frac{1}{64}\right) = \log_4\left(\frac{1}{4^3}\right) = \log_4(4^{-3}) = \boxed{-3}$$

5. Solve for x exactly:

(a) $\log_2(x) + \log_2(x-2) = 3$

$$\log_2(x) + \log_2(x-2) = 3 \Rightarrow 2^{\log_2(x) + \log_2(x-2)} = 2^3$$

$$\Rightarrow 2^{\log_2(x)}\, 2^{\log_2(x-2)} = 8 \Rightarrow x(x-2) = 8$$

$$\Rightarrow x^2 - 2x - 8 = 0$$

$$\Rightarrow (x-4)(x+2) = 0$$

$$\Rightarrow \boxed{x = 4, -2}$$

(b) $\ln x - \ln(x^2) = 5$.

$$\ln x - \ln x^2 = 5$$

$$\Rightarrow e^{\ln x - \ln x^2} = e^5$$

$$\Rightarrow e^{\ln x} \cdot e^{-\ln x^2} = e^5$$

$$\Rightarrow \frac{e^{\ln x}}{e^{\ln x^2}} = e^5 \Rightarrow \frac{x}{x^2} = e^5 \Rightarrow \frac{1}{x} = e^5$$

$$\Rightarrow \boxed{x = e^{-5}}$$

6. Write the following sets of numbers in interval notation

   (a) the open interval with endpoints 2 and 3 $(2, 3)$

   (b) $2 \leqslant x \leqslant 3$ $[2, 3]$

   (c) all real numbers x such that $x < -2$ and $x \geqslant 2$. $(-\infty, -2) \cup [2, \infty)$

7. Evaluate the following trigonometric functions exactly.

   (a) $\sin(\pi/3)$

   $= \frac{\sqrt{3}}{2}$

   (b) $\cos(\pi/3)$

   $= \frac{1}{2}$

   (c) $\sin(\pi)$

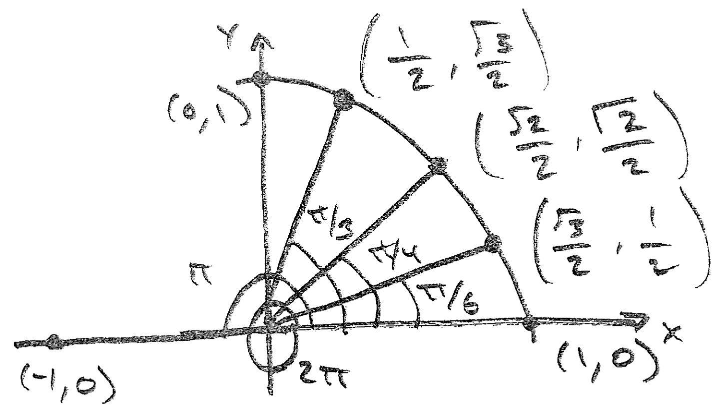
   $= 0$

   (d) $\sin(3\pi) = \sin(\pi)$

   $= 0$

   (e) $\cos(100\pi) = \cos(2\pi)$

   $= 1$

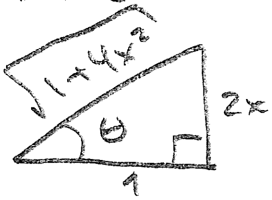


8. Simplify the following trigonemtric expression: $\sin(\arctan 2x)$.

Let $\theta = \arctan(2x)$

so

$\tan(\theta) = 2x$



$\Rightarrow \sin(\theta) = \sin(\arctan(2x))$

$= \frac{2x}{\sqrt{1+4x^2}}$

9. Find the inverse of the following function and state its domain: $f(x) = \frac{x}{1+2x}$.

Let $y = \frac{x}{1+2x} \Rightarrow \frac{1}{y} = \frac{1+2x}{x} \Rightarrow \frac{1}{y} = \frac{1}{x} + 2$

$\Rightarrow \frac{1}{y} - 2 = \frac{1}{x} \Rightarrow \frac{1-2y}{y} = \frac{1}{x}$ so $x = \frac{y}{1-2y}$.

Hence, $f^{-1}(x) = \frac{x}{1-2x}$ which has domain $(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

10. Below if a graph of $y = f(x)$. Sketch and label the following graphs.

(a) $y = f(x + 1)$ (b) $y = f(x - 1)$ (c) $y = f(2x)$ (d) $y = 2f(x)$ (e) $y = f(x) + 1$.