Calc β, BRIDGE 2018 Name: Key Due: Monday, July 16

Homework #7: Sharpening our tools

Note: Your work can only be assessed if it is legible.

1. **Basic derivatives.** Give the derivative of each of the following functions. You need not show your work.

$f(x) = c$, a constant

$f'(x) = 0$

$f(x) = x^n$, for a n a positive integer

$f'(x) = nx^{n-1}$

$f(x) = \sin x$

$f'(x) = \cos x$

$f(x) = \csc x$

$f'(x) = -\csc x \cot x$

$f(x) = \cos x$

$f'(x) = \sin x$

$f(x) = \sec x$

$f'(x) = \sec x \tan x$

$f(x) = \tan x$

$f'(x) = \sec^2 x$

$f(x) = \cot x$

$f'(x) = -\csc^2 x$

$f(x) = e^x$

$f'(x) = e^x$

$f(x) = a^x$, for $a > 0$

$f'(x) = a^x \cdot \ln a$

2. **Rules for differentiation.** Let $f(x)$ and $g(x)$ be differentiable functions. State the following rules of differentiation. (I have done the first one for you.)

   (a) State the *sum/difference rule.*

$$(f(x) \pm g(x))' = f'(x) \pm g'(x).$$

(b) State the *product rule.*

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

(c) State the *quotient rule.* (Be sure to include the extra assumption for the function in the denominator.)

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} \quad \text{where } g(x) \neq 0.$$

(d) State the *chain rule.*

$$(f(g(x)))' = f'(g(x)) \cdot g'(x).$$

3. **Working with the sum and difference rules.** Differentiate each function.

(a) $f(x) = x^5 + x^4 + x^3 + x^2 + x^1 + 1.$

$$f'(x) = 5x^4 + 4x^3 + 3x^2 + 2x + 1$$

(b) $f(x) = \cot x - \csc x.$

$$f'(x) = \csc x \cot x - \csc^2 x$$

(c) $f(x) = \sin x + x^5 - e^5.$

$$f'(x) = \cos x + 5x^4$$

(d) $f(x) = (x^5 - x^{1000}) + (5^x - 1000^x)$

$$f'(x) = 5x^4 - 1000x^{999} + 5^x \cdot \ln 5 - 1000^x \ln(1000)$$

4. **Working with the product and quotient rules.** Differentiate the following functions. You need not simplify.

(a) $f(x) = e^x \sin x$

$$f'(x) = e^x \sin x + e^x \cos x$$

(b) $f(x) = \dfrac{e^x}{\sin x}$

$$f'(x) = \frac{e^x \sin x - e^x \cos x}{\sin^2 x}$$

(c) $f(x) = (x^3 + 2x)e^x$

$$f'(x) = (3x^2+2)e^x + (x^3+2x)e^x$$

(d) $f(x) = \dfrac{e^x}{x^3 + 2x}$

$$f'(x) = \frac{e^x(x^3+2x) - e^x(3x^2+2)}{(x^3+2x)^2}$$

(e) $f(x) = \dfrac{e^x \sin x}{(x^3 + 2x)e^x}$.

Cancel e^x:

$$f'(x) = \frac{\cos x\,(x^3+2x) - \sin x\,(3x^2+2)}{(x^3+2x)^2}$$

-or-

use parts (a) and (c)

$$f'(x) = \frac{(e^x \sin x + e^x \cos x)(e^x(x^3+2x)) - (e^x \sin x)\left((3x^2+2)e^x + (x^3+2x)e^x\right)}{(x^3+2x)e^x}$$

(f) $f(x) = \dfrac{1 - xe^x}{x + e^x}$.

top derivative: $-e^x - xe^x$

bottom derivative: $1 + e^x$

so... $$f'(x) = \frac{(-e^x - xe^x)(x+e^x) - (1-xe^x)(1+e^x)}{(x+e^x)^2}$$

5. **Working with the chain rule.** Differentiate the following functions. If you use a result from a previous question, mention which question and part. You may use the fact that $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$ without justification.

(a) $f(x) = (x^4 + 3x^2 - 2)^5$

$$f'(x) = 5(x^4+3x^2-2)^4 \cdot (4x^3+6x)$$

(b) $f(x) = \tan(e^3 + x^3)$

$$f'(x) = \sec^2(e^3+x^3) \cdot 3x^2$$

(c) $f(x) = \cos(e^x) + e^{\cos x}$

$$f'(x) = -\sin e^x \cdot e^x + e^{\cos(x)}(-\sin x)$$

(d) $f(x) = 2^{x^2-1}$

$$f'(x) = \ln(2) \cdot 2^{x^2-1} \cdot (2x)$$

(e) $f(x) = \sqrt{1-2x}$

$$f'(x) = \frac{1}{2(\sqrt{1-2x})} \cdot (-2)$$

(f) $f(x) = (e^x + \sin x)^{256}$

$$f'(x) = 256(e^x+\sin x)^{255} \cdot (e^x + \cos x)$$

6. **Working with multiple rules.** Differentiate the following functions. If you use a result from a previous question, mention which question and part. You may use the fact that $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$ without justification.

(a) $f(x) = \sin^2 x + \cos^2 x$.

$f(x) = 1$ so $f'(x) = 0$

$\sin^2 x + \cos^2 x = 1$

(b) $f(x) = \sqrt{9\sin^2 x + 9\cos^2 x}$

$f(x) = \sqrt{9} = 3$ so $f'(x) = 1$

(c) $f(x) = \left(\frac{x^2+1}{x^2-1}\right)^3$

$$f'(x) = 3\left(\frac{x^2+1}{x^2-1}\right)^2 \cdot \left(\frac{x^2+1}{x^2-1}\right)' = 3\left(\frac{x^2+1}{x^2-1}\right)^2\left(\frac{2x(x^2-1) - 2x(x^2+1)}{(x^2-1)^2}\right)$$

(d) $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.

(For those that are curious, this function is actually the hyperbolic tangent function.)

$$f'(x) = \frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2}$$

(e) $f(x) = e^{t\sin 2t}$

$$f'(x) = e^{t \cdot \sin 2t} \cdot \left(\sin 2t + 2t \cos 2t\right)$$

(f) $f(x) = \sin(\sin(\sin x))$

$$f'(x) = \cos(\sin(\sin x))\,(\sin(\sin x))'$$

$(\sin x)'$

$$= \cos(\sin(\sin(x))) \cdot \cos(\sin x) \cdot \cos x$$

7. Given below is a table of values for differentiable functions $f(x)$ and $g(x)$ as well as their derivatives.

x	1	2	3	4
$f(x)$	3	6	6	11
$f'(x)$	1	0	0	1
$g(x)$	1	3	5	4
$g'(x)$	1	2	3	4

(a) If $a(x) = f(x) + 2g(x)$, what is $a'(1)$?

$a'(x) = f'(x) + 2g'(x)$ so $a'(1) = f'(1) + 2g'(1)$

$= 1 + 2(1) = 3$

(b) If $b(x) = f(x)g(x)$, what is $b'(2)$?

$b'(x) = f'(x)g(x) + f(x)g'(x)$

So $b'(2) = f'(2)g(2) + f(2)g'(2) = 0 + 6(2) = 12$

(c) If $c(x) = \dfrac{f(x)}{g(x)}$, what is $c'(3)$?

$c'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$ So $c'(3) = \dfrac{0 - 6(3)}{(5)^2}$

$= \dfrac{18}{25}$

(d) If $d(x) = f(g(x))$, what is $d(4)$?

$d(4) = f(g(4)) = f(4) = 11$

(e) What is $d'(4)$?

$d'(x) = f'(g(x))\, g'(x)$

So $d'(4) = f'(g(4)) \cdot g'(4) = f'(4) \cdot 4$

$= 1 \cdot 4 = 4$.