Calc β, BRIDGE 2018 Name: Key Due: Wednesday, July 11

Homework #5: The derivative of a function

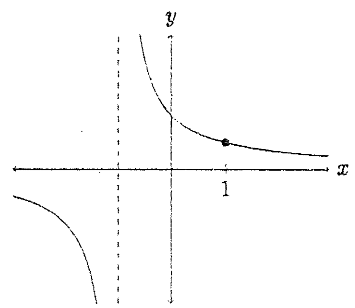
Note: Your work can only be assessed if it is legible. You must use the limit definition of the derivative. You do not need a calculator to complete this assignment.

1. Suppose $f(x)$ is a function such that $f(3) = 2$ and $f'(3) = 4$. Give an equation for the line tangent to the graph $y = f(x)$ at the point $(3, f(3))$.

Tangent line: $y - f(3) = f'(3)(x-3)$

So $\boxed{y - 2 = 4(x-3)}$

2. The function $f(x) = \dfrac{1}{x+1}$ is graphed below. Find $f'(1)$ and use it to give an equation of the tangent line to $y = f(x)$ at $x = 1$.



$$f'(1) = \lim_{x\to 1} \frac{f(x) - f(1)}{x-1} = \lim_{x\to 1} \frac{\frac{1}{x+1} - \frac{1}{1+1}}{x-1} = \lim_{x\to 1} \frac{1}{x-1}\left(\frac{2-(x+1)}{2(x+1)}\right)$$

$$= \lim_{x\to 1} \frac{1-x}{(x-1)(x+1)\cdot 2} = \lim_{x\to 1} \frac{-1}{2}\left(\frac{x-1}{(x-1)(x+1)}\right)$$

$$= \lim_{x\to 1} \frac{-1}{2}\left(\frac{1}{x+1}\right) = \boxed{\frac{-1}{4}}$$

Tangent line: $y - f(1) = f'(1)(x-1)$

So $\boxed{y - \frac{1}{2} = \frac{-1}{4}(x-1)}$

3. Find the derivative $f'(x)$ for each of the following functions.

(a) $f(x) = 4x^2 + 1$

$$f'(x) = \lim_{h\to 0} \frac{4(x+h)^2 + 1 - 4x^2 - 1}{h} = \lim_{h\to 0} \frac{4x^2 + 8xh + 4h^2 - 4x^2}{h}$$

$$= \lim_{h\to 0} 8x + 4h = \boxed{8x}$$

(b) $f(x) = \sqrt{2x}$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{2x+2h} - \sqrt{2x}}{h}\left(\frac{\sqrt{2x+2h} + \sqrt{2x}}{\sqrt{2x+2h} + \sqrt{2x}}\right) = \lim_{h\to 0} \frac{2x+2h-2x}{h(\sqrt{2x+2h} + \sqrt{2x})}$$

$$= \lim_{h\to 0} \frac{2}{\sqrt{2x+2h} + \sqrt{2x}} = \frac{2}{2\sqrt{2x}} = \boxed{\frac{1}{\sqrt{2x}}}$$

(c) $f(x) = \dfrac{1-x}{2+x}$

$$f'(x) = \lim_{h\to 0}\left(\frac{1-(x+h)}{2+(x+h)} - \frac{1-x}{2+x}\right)\cdot\frac{1}{h} = \lim_{h\to 0} \frac{1}{h}\left(\frac{(1-(x+h))(2+x) - (1-x)(2+x+h)}{(2+x+h)(2+x)}\right)$$

after some algebra

$$= \lim_{h\to 0} \frac{-3}{x^2+4x+4+xh+2h} = \boxed{\frac{-3}{x^2+4x+4} = \frac{-3}{(x+2)^2}}$$

(d) $f(x) = mx + b$ where m and b are arbitrary constants.

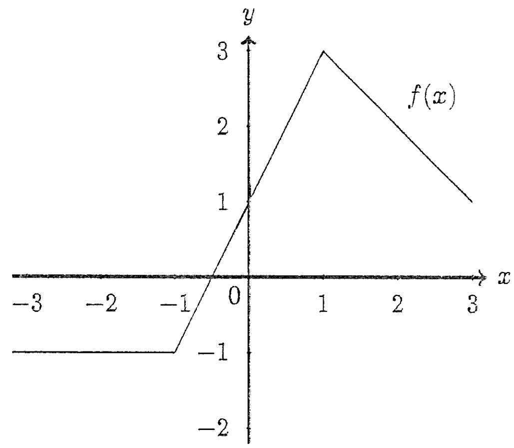
$$f'(x) = \lim_{h\to 0} \frac{m(x+h)+b-(mx+b)}{h} = \lim_{h\to 0} \frac{\cancel{mx} + m\cancel{h} + \cancel{b} - \cancel{mx} - \cancel{b}}{\cancel{h}}$$

$$= \lim_{h\to 0} m = \boxed{m}$$

Every ~~g~~ line tangent to $y = mx+b$ has slope m!

That's because every tangent line is $y=mx+b$ itself!

4. The graph of $y = f(x)$ is pictured below.



(a) Compute each derivative below. If a derivative does not exist, write DNE.

i. $f'(-2)$

$= 0$

ii. $f'(1)$

DNE

iii. $f'(-1)$

DNE

iv. $f'(2)$

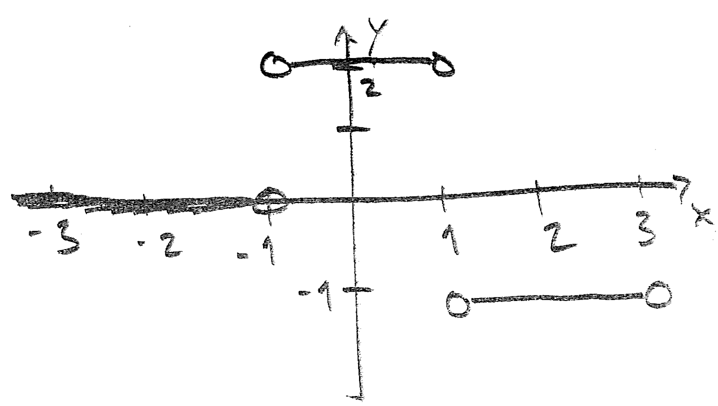
$= -1$

v. $f'(0)$

$= 2$

[redacted]

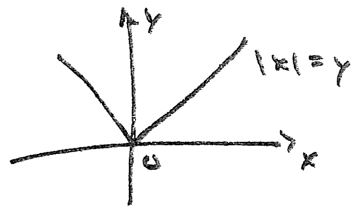
(b) Sketch a graph of the derivative $f'(x)$ for $-3 \leqslant x \leqslant 3$.



5. In class we mentioned that if a function is differentiable at $x = a$ then it is also continuous there. With that in mind, consider the following statements.

(a) T/F (with justification) A function that is continuous at a is also differentiable at a.

$\boxed{F}$ $f(x) = |x|$ is continous at $x=0$ but not differentiable there due to a cusp.



(b) T/F (with justification) If $f'(2)$ exists, then $\lim_{x\to 2} f(x) = f(2)$.

$\boxed{T}$ If $f(x)$ is differentiable at 2, then $f(x)$ is continuous at 2, there fore $\lim_{x\to 2} f(x) = f(2)$.

6. **Bonus:** Give the name of a function which is continuous at every point but is differentiable at no point.
Hint: Use Google.

"The Weierstrass Function"