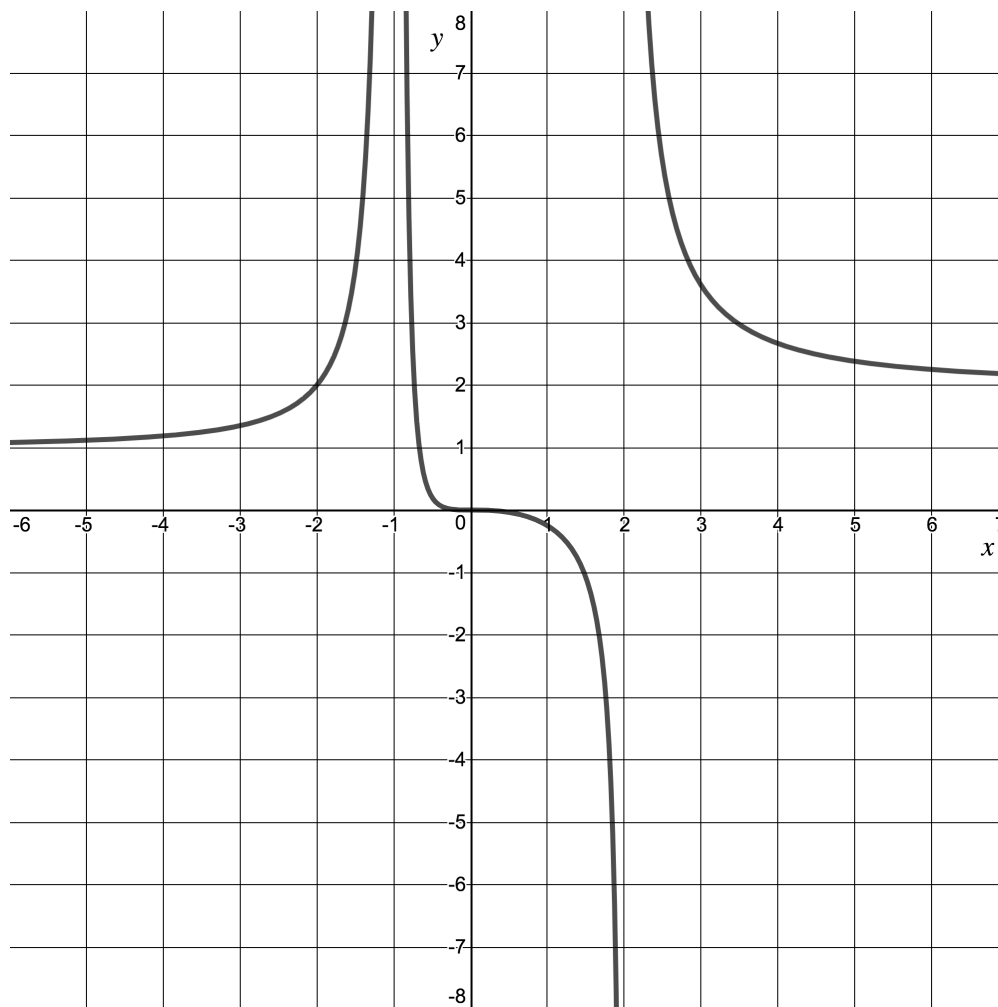

Homework #4: Limits and infinity

Note: Your work can only be assessed if it is legible. You do not need a calculator to complete this assignment.

1. Given below is the graph of a function $f(x)$.



- (a) Specify the vertical asymptotes of $y = f(x)$ and justify your statements with an appropriate statement regarding limits.
- (b) Specify the horizontal asymptotes of $y = f(x)$ and justify your statements with an appropriate statement regarding limits.
- (c) *Bonus:* Give an expression for a possible function $f(x)$ which might have this graph.

2. Compute the following limits or explain why they do not exist (and if it approaches ∞ or $-\infty$.)

(a) $\lim_{x \rightarrow 1^+} \frac{x-2}{x-1}$

(b) $\lim_{x \rightarrow \infty} \frac{2x+3}{6x-7}$

(c) $\lim_{x \rightarrow 0} \frac{1}{x} - \frac{1}{x^2}$

(d) $\lim_{x \rightarrow \infty} \frac{x^3}{\sqrt{6x^4-1}}$

(e) $\lim_{x \rightarrow -\infty} \frac{4x^3 + 6x^2 - 2}{2x^3 - 4x + 5}$

(f) $\lim_{x \rightarrow \infty} \sqrt{x^2 + 1} - x$.

Hint: Multiply the expression by 1 in the form of the conjugate radical.

3. (a) T/F (with justification) The line $x = 1$ is a vertical asymptote of the graph $y = \frac{x^2 - 1}{x^2 - 2x + 1}$.

(b) T/F (with justification) The line $x = 1$ is a vertical asymptote of the graph $y = \frac{x^2 - 2x + 1}{x^2 - 1}$.

4. Consider the function $f(x) = \frac{x}{\sqrt{4 + 2x^2}}$.

(a) Compute $\lim_{x \rightarrow \infty} f(x)$.

(b) Compute $\lim_{x \rightarrow -\infty} f(x)$.

(c) What are the horizontal asymptotes of $y = f(x)$?

(d) Does $f(x)$ have any vertical asymptotes? Justify your answer.

(e) Based upon your previous work, sketch a possible graph of $f(x)$.