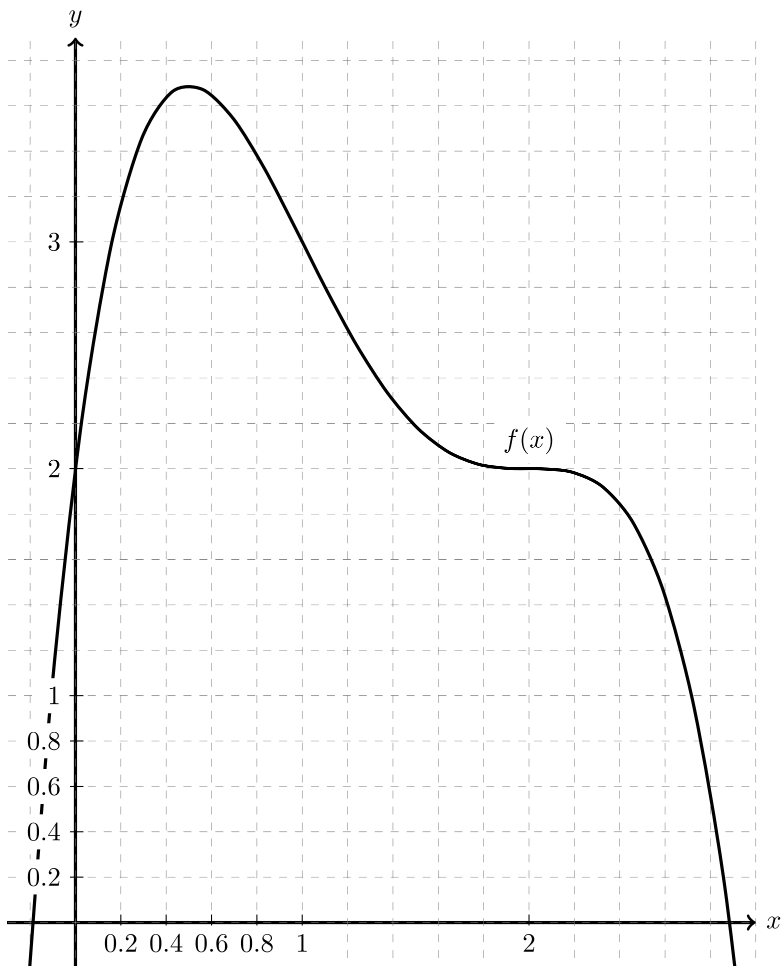
Calc β, BRIDGE 2018 **Name:** __________________________ **Due:** Monday, July 9

Homework #3: The precise definition of the limit

Note: Your work can only be assessed if it is legible. You must show all of you work on all problems save 1 and 5. You do not need a calculator to complete this assignment.

1. Consider the function $f(x)$ whose graph is given below. It is clear that $\lim_{x\to 1} f(x) = 3$. Hence, given any $\varepsilon > 0$, we should be able to find a $\delta > 0$ such that $|f(x) - 3| < \varepsilon$ when $|x - 1| < \delta$.



Let $\varepsilon = 0.4$. Find an appropriate δ such that $|x - 2| < \delta$ implies $|f(x) - 3| < \varepsilon = 0.4$.

2. Let $f(x) = 2x + 3$ and $\varepsilon = 0.5$. We consider $\lim_{x\to 0} 2x + 3$.

   (a) Find $\lim_{x\to 0} 2x + 3$.

   (b) Let $L = \lim_{x\to 0} 2x + 3$, the number you found in the previous part. Find a number $\delta > 0$ such that if $|x - 0| < \delta$ then $|f(x) - L| < \varepsilon$.

3. Let $f(x) = x^2 - 2x + 6$ and note $\lim_{x\to 1} x^2 - 2x + 6 = 5$. Here we will work through verifying this fact. To do this, we will need to find for *any* $\varepsilon > 0$ a related $\delta > 0$ such that

$$\text{if } |x - 1| < \delta \text{ then } |f(x) - 5| < \varepsilon.$$

   (a) We begin by studying a few concrete examples. Let $\varepsilon = \dfrac{1}{4}$. Find an appropriate value for δ.

   (b) Repeat for $\varepsilon = \dfrac{1}{25}$.

   (c) Repeat for $\varepsilon = \dfrac{1}{100}$.

(d) Is there a common relationship between each ε and δ you have found?

(e) Let $\varepsilon > 0$ be arbitrary. Find a $\delta > 0$ (in terms of ε) that guarantees $|f(x) - 5| < \varepsilon$ when $|x - 1| < \delta$.

(f) **Bonus:**

Use the previous part to *prove* $\lim_{x \to 1} x^2 - 2x + 6 = 5$.

(To get started, write something like "*Let ε be any positive number. Then choose δ to be . . . .*")

4. **Bonus:** Using the precise definition of the limit, prove that $\lim_{x \to -3} 1 - 4x = 13$.