Calc β, BRIDGE 2018 **Name:** ____________________________ **Due:** Tuesday, July 31

Homework #16: The fundamental theorem of calculus

Note: Your work can only be assessed if it is legible.

1. Find the derivative of each of the following functions.

   (a) $\displaystyle\int_0^x \frac{1}{1+t^5}\,dt$

   (b) $\displaystyle\int_2^x \sin(e^{2t})\,dt$

   (c) $\displaystyle\int_1^{e^x} \ln t\,dt$

   (d) $\displaystyle\int_0^{x^3} t^2\cos t\,dt$

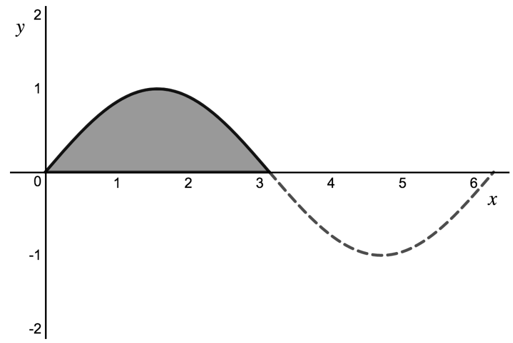
2. Evaluate the following definite integrals using the fundamental theorem of calculus.

(a) $\int_{-1}^{1} x^{20} + x^{18}\, dx$

(b) $\int_{0}^{1} x^{e} + e^{x}\, dx$

(c) $\int_{0}^{1} e^{x+1}\, dx$

3. Find the area bound by "one hump" of $\sin x$. That is, find the area shown below. The plotted graph is $y = \sin x$ on the interval $[0, 2\pi]$.



4. On the previous homework, you estimated the area under the curve $y = 4 - x^2$ over the interval $[0, 2]$. Use the fundamental theorem of calculus to compute the exact area.

5. (a) Let $A_0(x) = \int_0^x 1 - t^2\,dt$, $A_1(x) = \int_1^x 1 - t^2\,dt$, and $A_2(x) = \int_2^x 1 - t^2\,dt$.
Compute these explicitly in terms of x using part 2 of the fundamental theorem of calculus.

(b) Over the interval $[0, 2]$, use your answers in part (a) to sketch the graphs of $y = A_0(x)$, $y = A_1(x)$, and $y = A_2(x)$ on the same set of axes.

(c) How are the three graphs in part (a) related to each other? In particular, what does part 1 of the fundamental theorem of calculus tell you about the graphs in part (a)?

(d) On a graph of $y = 1 - t^2$, for $0 \leqslant t \leqslant 2$, shade the region with signed area $A_0(1.5)$. Indicate with $+$ and $-$ which area counts positively and which negatively.

6. T/F (with justification) The function $F(x) = \int_0^x \cos(t^2)\, dt$ is an antiderivative of $\cos(x^2)$.

7. T/F (with justification) $\int_{-2}^{2} x^{-4}\, dx = \left.\frac{x^{-3}}{-3}\right|_{-2}^{2} = \frac{-1}{12}$.

8. Evaluate each indefinite integral.

(a) $\int x^3 + \frac{1}{x^2}\,dx$

(b) $\int \frac{x^5 - 2\sqrt{x^3}}{x}\,dx$

(c) $\int \frac{\sin x}{\cos^2 x}\,dx$. (*Hint:* $\frac{1}{\cos x} = \sec x$.)

9. Verify by differentiation that the following equation is true:

$$\int \frac{1}{x^2\sqrt{1+x^2}}\,dx = \frac{-\sqrt{1+x^2}}{x} + C.$$

In class, we mentioned that the definite integral of $f'(x)$ over an interval $[a, b]$ represents the *net change* in $f(x)$ due to the fundamental theorem of calculus:

$$\int_a^b f'(x)\,dx = f(b) - f(a).$$

To see this in a scientific context, note the following implications:

- If $V(t)$ is the volume of water in a reservoir at time t, then its derivative $V'(t)$ is the rate at which water flows into the reservoir at time t. So

  $$\int_{t_1}^{t_2} V'(t)\,dt = V(t_2) - V(t_1)$$

  is the change in the amount of water in the reservoir between time t_1 and time t_2.

- If $[C](t)$ is the concentration of the product of a chemical reaction at time t, the rate of reaction is the derivative $d[C]/dt$. So

  $$\int_{t_1}^{t_2} \frac{d[C]}{dt}\,dt = [C](t_2) - [C](t_1)$$

  is the change in the concentration of $[C]$ from time t_1 to t_2.

- If the mass of a rod measured from the left end to a point x is $m(x)$, then the linear density is $\rho(x) = m'(x)$. So

  $$\int_a^b \rho(x)\,dx = m(b) - m(a)$$

  is the mass of the segment of the rod that lies between $x = a$ and $x = b$.

- If the rate of growth of a population is dn/dt, then

  $$\int_{t_1}^{t_2} \frac{dn}{dt}\,dt = n(t_2) - n(t_1)$$

  is the net change in population during the time period from t_1 to t_2.

- If an object moves along a straight line with position function $s(t)$, then its velocity is $v(t) = s'(t)$, so

  $$\int_{t_1}^{t_2} v(t)\,dt = s(t_2) - s(t_1)$$

  is the net change of position, or *displacement,* of the particle during the time period from t_1 to t_2.

- The acceleration of the object is $a(t) = v'(t)$, so

  $$\int_{t_1}^{t_2} a(t)\,dt = v(t_2) - v(t_1)$$

  is the net change in velocity from time t_1 to t_2.

10. Water is released into a tank at the rate $r(t) = 5 + \sqrt{t}$ ft^3/min at time t (in minutes). At $t = 1$ minute, there is 12 ft^3 of water in the tank.

(a) Evaluate $\int_1^9 r(t)\,dt$.

(b) In the context given above, what does the value in part (a) tell us?

(c) Determine the volume of water in the tank at time $t = 9$ minutes.