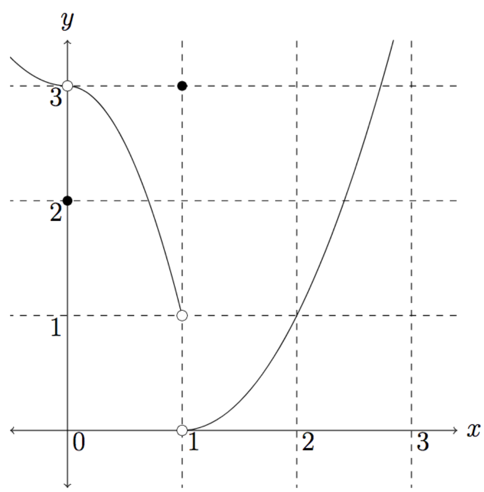
Calc β, BRIDGE 2018 **Name:** ____________________________ **Due:** Thursday, July 5

Homework #2: Limits and continuity

Note: Your work can only be assessed if it is legible. You must show all of you work on all problems save 1 and 5. You do not need a calculator to complete this assignment.

1. The graph of $y = f(x)$ is below. Use it to compute each limit or explain why it doesn't exist.



(a) $\lim_{x\to 0^-} f(x)$

(b) $\lim_{x\to 1^-} f(x)$

(c) $\lim_{x\to 2^-} f(x)$

(d) $\lim_{x\to 0^+} f(x)$

(e) $\lim_{x\to 1^+} f(x)$

(f) $\lim_{x\to 2^+} f(x)$

(g) $\lim_{x\to 0} f(x)$

(h) $\lim_{x\to 1} f(x)$

(i) $\lim_{x\to 2} f(x)$

(j) $f(0)$

(k) $f(1)$

(l) $f(2)$

2. T/F (with justification) If $\lim_{x\to 2} g(x) = 0$ and $\lim_{x\to 2} h(x) = 0$ then $\lim_{x\to 2} \dfrac{g(x)}{h(x)}$ does not exist.

3. Evaluate the following limits exactly using algebra and limit laws.

(a) $\lim_{x\to 9} \dfrac{x-9}{\sqrt{x}-3}$

(b) $\lim_{x\to -4} \dfrac{x^2+4x}{x^2+3x-4}$

4. The squeeze theorem will prove valuable in this problem.

(a) Evaluate $\lim_{x\to 1}(x-1)^4 \cos\left(\dfrac{1}{1-x}\right)$.

(b) **Bonus:** Let

$$f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational,} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

Show that $\lim_{x\to 0} f(x) = 0$.

5. Let

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1, \\ 4 & \text{if } x = 1, \\ x + 2 & \text{if } 1 < x \leqslant 2, \\ 6 - x & \text{if } x > 2. \end{cases}$$

Evaluate the following limits if they exist. If a limit does not exist, write DNE.

(a) $\lim_{x \to 1^-} f(x)$

(b) $\lim_{x \to 1^+} f(x)$

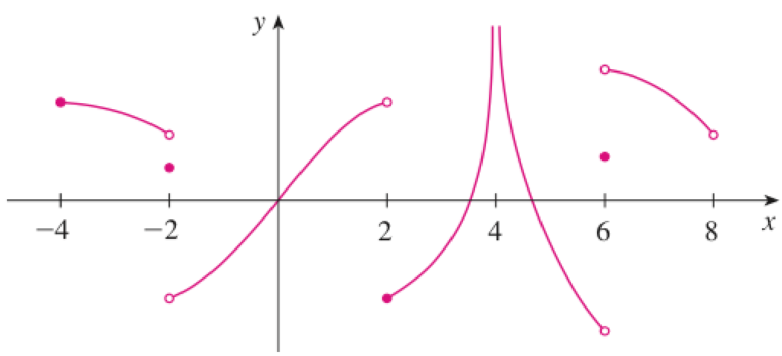
(c) $\lim_{x \to 1} f(x)$

(g) $\lim_{x \to 2^-} f(x)$

(h) $\lim_{x \to 2^+} f(x)$

(i) $\lim_{x \to 2} f(x)$

6. The graph of a function $g(x)$ is given below. State the intervals on which g is continuous.



7. Is

$$f(x) = \begin{cases} \sin x & \text{if } x \leqslant 0, \\ 1 + \cos x & \text{if } x > 0. \end{cases}$$

continuous on the interval $(-1, 1)$?

8. For what value of the constant c is the function f continuous on the entire real line $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2, \\ x^3 - cx & \text{if } x \geqslant 2. \end{cases}$$

9. Use the intermediate value theorem to show that there is a solution to $x - \sqrt{x} - \ln x = 0$ on the interval $[2, 3]$. Explain your reasoning.

Hint: You may find the following facts helpful. $\sqrt{2} \geqslant 1.41 \quad \ln 2 \geqslant 0.69$ $\sqrt{3} \leqslant 1.74 \quad \ln 3 \leqslant 1.1$